Game Theory and Its Applications to Communications

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Outline of the Presentation

- Introduction

- Game theory: basic definitions and notions

- Specific applications of non cooperative game theory to wireless ad-hoc networks

Introduction

- Most human activities are limited by the **finite availability of resources**.

- In communications engineering the fundamental resources are represented by **bandwidth** (i.e., **spectrum**) and **power**.

- **Power** is needed to overcome noise at the receiver, and it is basically limited because of regulations and due to limitations on storage (i.e., batteries).

- **Spectrum** is fundamentally in shortage, because the part of the electromagnetic spectrum exploitable for radio wave propagation is rather limited.

Introduction

licensed (dedicated) spectrum (very expensive; e.g., see spectrum license auctions for 3G telephony)

Spectrum

unlicensed spectrum (it can be overcrowded at some locations and so cluttered with interference that systems provide a very poor grade of service or even cease to work)

Introduction

• **Game theory** is a branch of mathematics and provides a set of tools for analyzing resource conflicts, or more generally, optimization problems with multiple conflicting objective functions.

• For this reason, it can be used to analyse all those communication problems in which the finiteness of spectrum and power creates a **resource conflict**.

Game theory: basic definitions and notions

• Generally speaking, a **static *N* player game** *G* **in strategic** (or **normal**) **form** is a triplet

$$G = \left(\left\{1, 2, ..., N\right\}, S, \mathbf{u}\right)$$

- First, there are the parties $\left\{1, 2, ..., N\right\}$ involved in the resource conflict; these will be called **players**;

- Second, the actions or moves that can be taken by the players are called **strategies**; these belong to the **strategy space** *S*, which collects all the possible combinations of actions by each player;

- The third element consists of the **vector utility function** $\mathbf{u} = \left[u_1, u_2, ..., u_N\right]$, collecting the payoffs obtained by the players; such payoffs will depend on the strategies selected by the players.

Game theory: basic definitions and notions

$S = A_1 \times A_2 \ldots. \times A_N$ where A_n is the set of actions for the *n*-th player

$u_n(a_1, a_2, \ldots, a_n): S \to \Re$ is the **utility** of the *n*-th player when the strategy vector $(a_1, a_2, \ldots, a_n)$ has been played

The players have chosen the actions $a_1, a_2, \ldots, a_n$

Player *n* receives a payoff equal to $u_n(a_1, a_2, \ldots, a_n)$

Game theory: basic definitions and notions

• **Utility** is a measure of how much something is worth to someone.

• The players are assumed to be **rational** (i.e., each player does what is best for him) and **selfish**.

• **Their objective is maximizing their utility in the game**.

• In a more general context, utility may represent real or monetary values and it may be measured in arbitrary units. **Utility is not necessarily linear in the amount owned**. For instance, utility of money is often argued to be logarithmic in the owned amount x, i.e.

$$\text{utility} = \log(x+1).$$

• One euro has much more worth for someone who has nothing than for someone who already owns a lot of money!

Game theory: basic definitions and notions

• **Static** game $\Rightarrow$ the game is played in one shot

• Game with **complete information** $\Rightarrow$ every player knows the triplet G

• In **communication games**, the players, $\{1,2,...,N\}$, are usually the transmitters and the receivers sharing a wireless channel.

• In a **wireless communications** application, a *set of strategies* may refer to which spectral band a user is transmitting in, or how much power a user spends. The utilities may be the *rates* at the receivers.

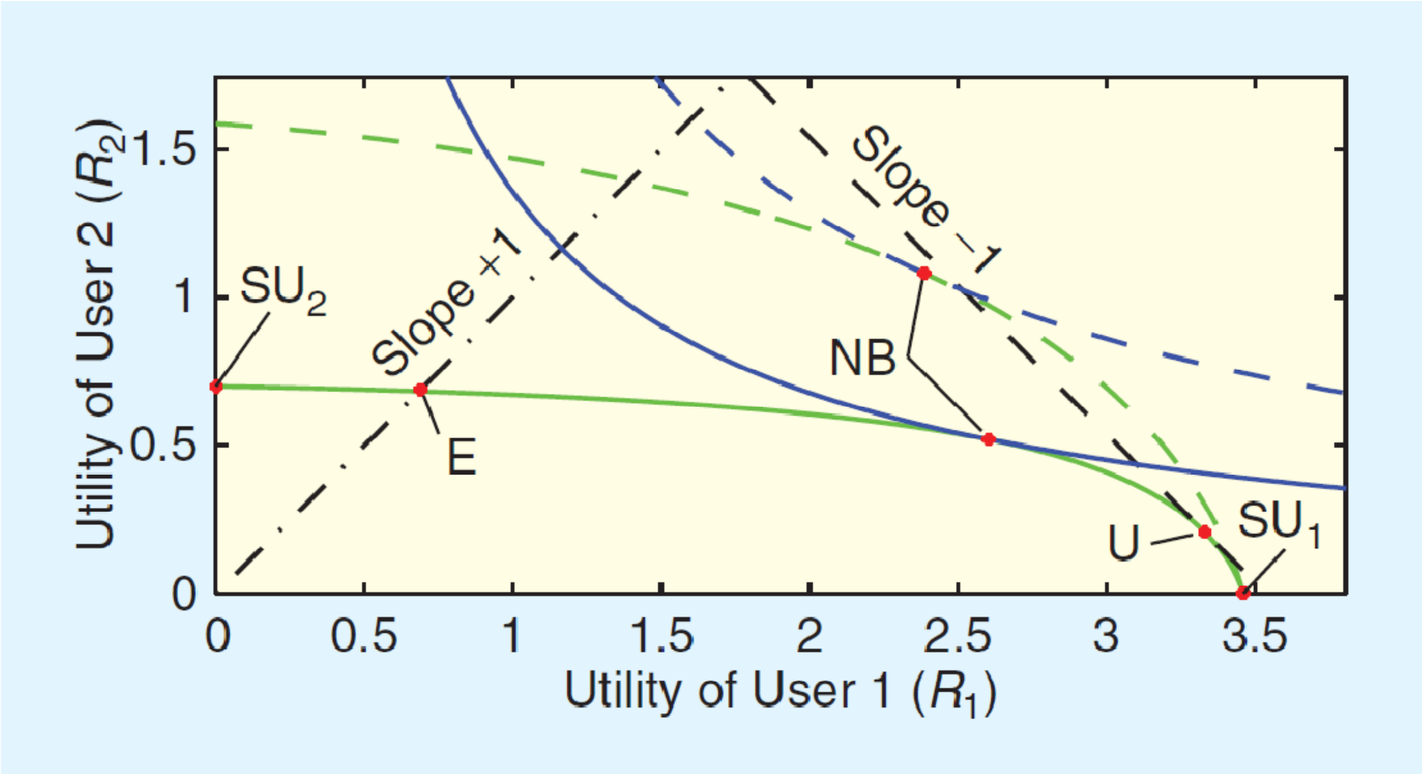
Game theory: basic definitions and notions

• When increasing someone else's utility means decreasing your own, we say that we have a **conflict**. All resource allocation problems are conflicts in this sense.

• The set of all possible outcomes of a conflict is called the **utility region**.

• The northeast boundary of the utility region is called the **Pareto boundary**, because it consists of **Pareto optimal operating points**. These are points at which increasing the utility for one of the players necessarily must decrease the utility for the other.

Example 1 - Example of utility region and some interesting points



Pareto boundary: solid green curve
Utility region: region below the solid green curve

Game theory: basic definitions and notions

- In the previous figure the following special operating points can be easily identified:

1. The **utilitarian point** (U)

$$R_1 + R_2 \quad \text{is maximized}$$

This is the point where a straight line with slope -1 touches the Pareto boundary (in communications this is usually dubbed the **“sum-rate” point**).

2. The **egalitarian point** (E)

$$\min\left(R_1, R_2\right) \quad \text{is maximized}$$

This point is represented by the intersection between the Pareto boundary and a straight line having slope +1 and passing through the origin.

3. **Single-user** (SU_1 , SU_2)

$$R_2 = 0 \quad \text{and} \quad R_1 = 0$$

respectively.

Game theory: basic definitions and notions

• Games are usually represented in one of the following two forms: the *normal form* and the *extensive form.*

• The **normal form** game for two players is represented as a **bimatrix**, as shown in the following **Example** (from A. B. MacKenzie and S. B. Wicker, “Game Theory and the Design of Self-Configuring, Adaptive Wireless Newtworks”, IEEE Commun. Mag., Nov. 2001).

• An **extensive form** game is depicted as a **tree**, where each node represents a decision point for one of the players. The normal form is easier to analyze, but the extensive form captures the structure of a real game in time.

• Example 2 – Battle of the Sexes

• The following Figure shows a normal form version of a coordination game known as “The Battle of the Sexes”.

• In this game, Abelard and Eloise would like to attend a concert together. Unfortunately they have different tastes in music: Abelard would prefer to attend a Rolling Stones concert, while Eloise would prefer an opera by Mozart. Both, however, would rather go to either performance together than attend their favorite concert alone.

• In the normal form version of this game, the **rows** represent **Abelard**’s choice of strategies, while the **columns** represent **Eloise**’s choices. In this case the same strategies are available to both, although this need not be the case. Given strategy selections by both players, we go to the corresponding bimatrix element and read off the payoffs for the two players, with the row player (Abelard) getting the first number and the column player (Eloise) the second. The higher number represents the greater payoff.

Game theory: basic definitions and notions

Eloise

		Rolling Stones	Mozart
Abelard	Rolling Stones	**2, 1**	**0, 0**
	Mozart	**0, 0**	**1, 2**

• The game proceeds by having each player **simultaneously** announce their choices. In this simple game, we assume that each player is stuck with whatever choice he or she makes.

Game theory: basic definitions and notions

• Suppose that Abelard and Eloise both choose to go to the Rolling Stones concert. Note that even if one of them *could* change their choice of strategy at this point, neither would (because the payoff would decrease).

• We need not limit Abelard and Eloise to the choice of one strategy or the other. Such a choice is called a **pure** strategy, in contrast to a **mixed** strategy which is represented to a probability distribution on a player's available pure strategies.

• For example, Eloise may decide that she will attend each concert with probability 0.5. To obtain the payoff when one or both players choose a mixed strategy, we simply compute the expected value of each player's payoff. For instance, suppose that both players choose a *50/50* mixed strategy. An expected value analysis shows that each player can expect a payoff of 0.75.

• Let us focus now on a specific communication problem.

Example 3 – Resource conflicts on the Gaussian interference channel

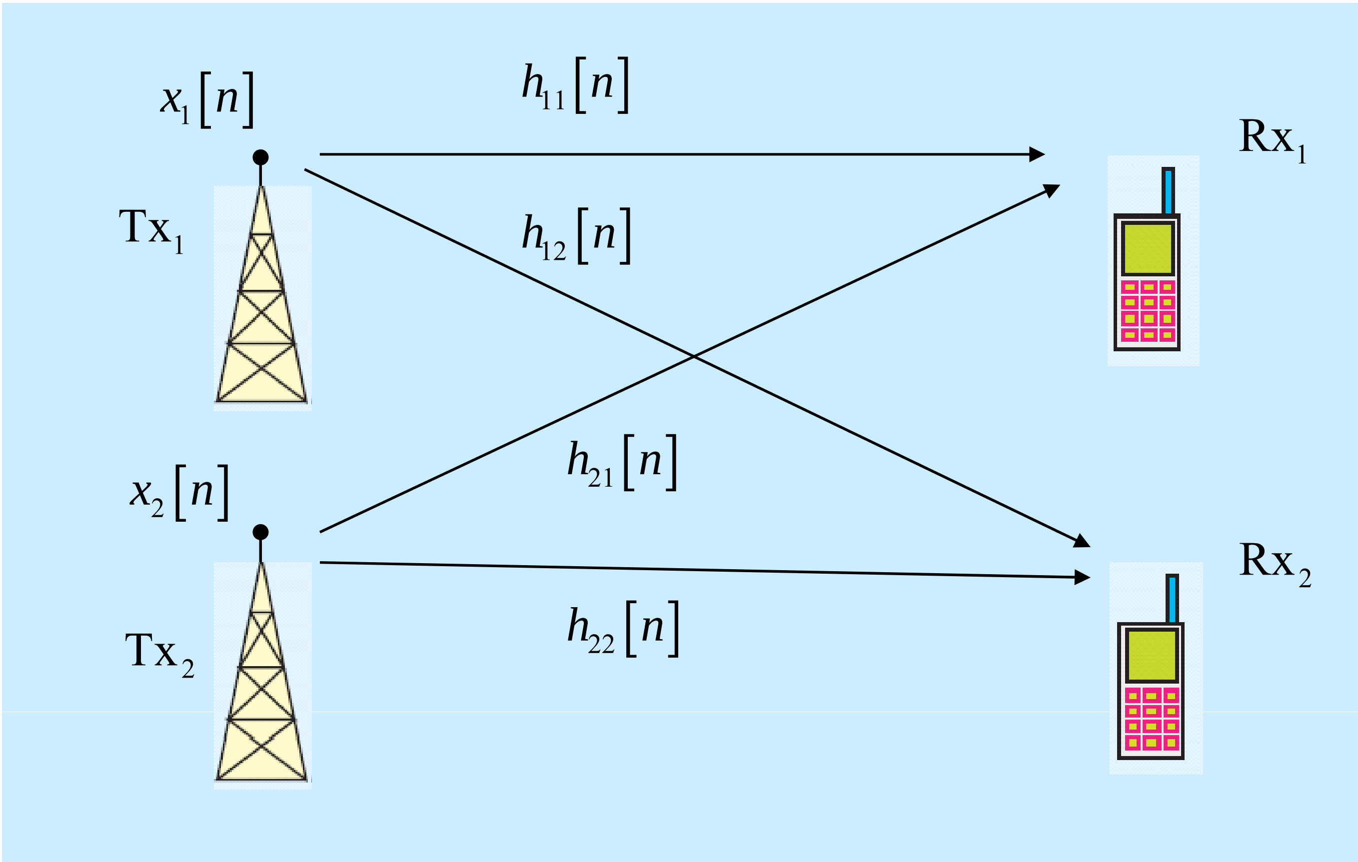
• We consider the so-called *single-input, single-output* (SISO) interference channel; the corresponding resource allocation problem is formulated as a **power game** (involving two players).

• Consider two transmitter-receiver pairs, TX1 -> RX1 and TX2 ->RX2, that operate in the same spectral band, so generating mutual interference.

• Suppose that the first (second) system transmits the signal $x_1[n]$ ($x_2[n]$) using the power P_1 (P_2). The signals at the two receivers can then be modeled as

$$\begin{aligned} y_1[n] &= h_{11}[n]x_1[n] + h_{21}[n]x_2[n] + n_1[n] \\ y_2[n] &= h_{12}[n]x_1[n] + h_{22}[n]x_2[n] + n_2[n] \end{aligned} \qquad (1)$$

Game theory: basic definitions and notions



From E. G. Larsson, E. A. Jorswieck, J. Lindblom, and R. Mochaourab, "**Game Theory and the Flat-Fading Gaussian Interference Channel**", IEEE Sig. Proc. Mag., Sep. 2009.

Game theory: basic definitions and notions

• The signal model (1) characterizes an **interference channel** (IFC).

• The IFC is a complicated topic: the capacity of this channel is unknown.

• However, it is known that:

- If the **interference** at any of the receivers is **strong** enough to be decoded by treating the desired signal as noise when doing this decoding, then the following scheme is **optimal**: the receiver first decodes the interference, and then subtracts the decoded interference from the received signal to obtain interference-free data (*serial interference cancellation*, SIC).

- Conversely, if the **interference** at any receiver is **very weak**, then it is optimal to just treat it as additional additive noise. In the following we assume that the receivers treat the interference as noise.

Game theory: basic definitions and notions

• The sequences $n_1[n]$ and $n_2[n]$ are samples of a zero mean circularly symmetric complex Gaussian noise process with variance σ^2

• The assumption made on the powers implies that $E\left\{\left|x_i[n]\right|^2\right\} = P_i$

• **The systems compete with each other for resources**, because if one of the transmitters increases its transmit power in an attempt to improve performance (SINR at its receiver), then it will simultaneously increase the amount of interference generated to the other system. The "strategy" space for the two systems consists of **how much power to spend** (P_1, P_2) and **during what fraction of the available time to transmit**.

• The problem is well posed if we specify *constraints* on the power that can be spent, say

$$P_1 \le \bar{P}_1,\ P_2 \le \bar{P}_2$$

Game theory: basic definitions and notions

- In the following we make the following **assumptions**:

1. Both systems operate under the same power constraint; this means that

$$\overline{P}_1 = \overline{P}_2 = \overline{P} \text{ for some } \overline{P}$$

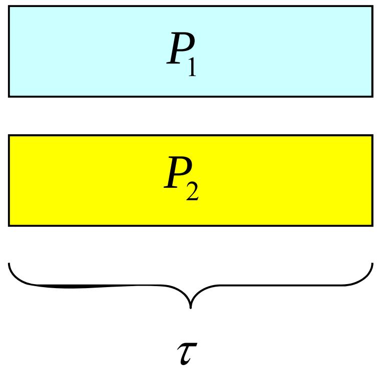
2. The receivers treat the interference as noise (the main reason for this is that a decode-and-subtract-interference strategy would require all systems to know the coding and modulation formats of all other systems; this is a questionable assumption).

3. The transmitters use capacity-achieving coding, so that Shannon's log(1+SNR) formula can be used.

Game theory: basic definitions and notions

- **First scenario** - Both systems transmit continuously with powers P_1 , P_2 (continuous interference)

- The rates at the receivers are



$$R_1 = \log_2\left(1 + \frac{P_1 |h_{11}|^2}{P_2 |h_{21}|^2 + \sigma^2}\right) \qquad R_2 = \log_2\left(1 + \frac{P_2 |h_{22}|^2}{P_1 |h_{12}|^2 + \sigma^2}\right)$$

- The rate (**utility**) region can be defined as

$$\mathrm{R} = \bigcup_{P_1 \le \bar{P}, P_2 \le \bar{P}} (R_1, R_2)$$

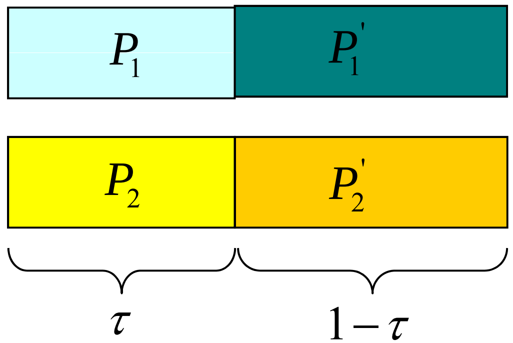
- Clearly, to achieve points on the boundary at least one of transmitters must use maximum power.

$$G = \left(\{1,2\}, \left[0, \bar{P}\right]^2, \{R_1, R_2\}\right)$$

Game theory: basic definitions and notions

• **Second scenario** - To achieve points outside the region R one can use a technique called **time-sharing**. This amounts to splitting the available time into two subslots of relative lengths τ and 1 - τ, where $0 \le \tau \le 1$ and use two different pairs of transmit powers.

• For a given τ, the achievable rate pair becomes



$$\left(\tau R_1 + (1-\tau) R_1', \tau R_2 + (1-\tau) R_2'\right)$$

where

$$R_1' = \log_2\left(1 + \frac{P_1' |h_{11}|^2}{P_2' |h_{21}|^2 + \sigma^2}\right)$$

$$R_2' = \log_2\left(1 + \frac{P_2' |h_{22}|^2}{P_1' |h_{12}|^2 + \sigma^2}\right)$$

$$G = \left(\{1,2\}, \left[0, \bar{P}\right]^2 \times [0,1], \{R_1, R_2\}\right)$$

• The power constraint

$$P_1 \le \overline{P}_1,\ P_2 \le \overline{P}_2$$

can be interpreted either as a **peak constraint**, or as a limit on the **average transmit power** (when the transmission is intermittent). Depending on how the power constraint is interpreted, two rate regions will emerge in the case of time sharing.

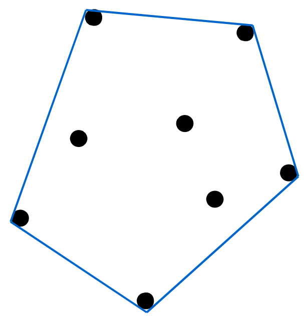
• We will assume that the **peak power is constrained**. Then, the resulting rate region in the presence of time-sharing is

$$\overline{\mathrm{R}} = \bigcup_{\substack{0 \le \tau \le 1 \\ 0 \le P_1, P_1' \le \overline{P}, 0 \le P_2, P_2' \le \overline{P}}} \left(\tau R_1 + (1-\tau) R_1', \tau R_2 + (1-\tau) R_2'\right)$$

Game theory: basic definitions and notions

• It can be shown that $\bar{\mathrm{R}}$ as the **convex hull** of R. Therefore, time-sharing with a peak power constraint leads to a convexification of the rate region R.

*In mathematics, the **convex hull** or **convex envelope** for a set of points *X* in a real vector space V is the **minimal convex set containing** *X*.

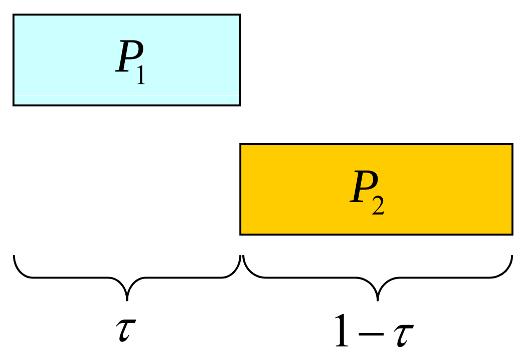


Game theory: basic definitions and notions

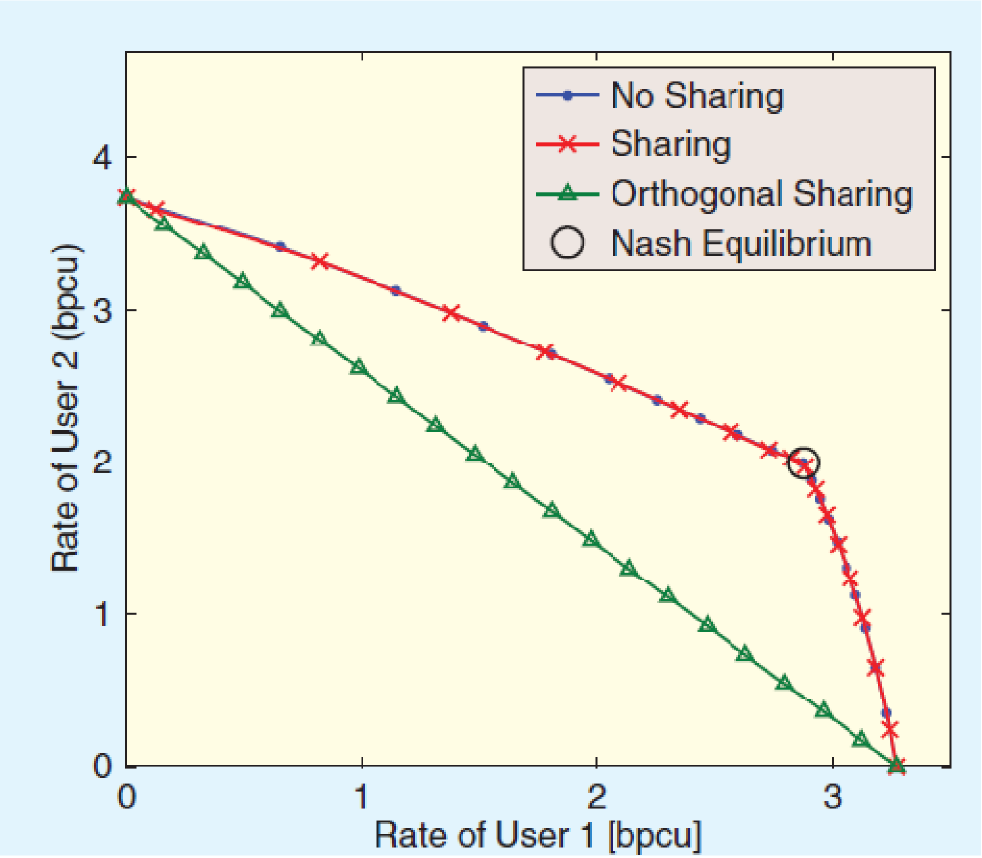
• **Third scenario -** A special case of time-sharing is when the strategies are chosen such that the systems do not produce any interference to each other.

• In practice, this can be simply accomplished by separating the systems in time or frequency. In accordance with most literature, we refer to this case as “**orthogonal transmission**”. In this scenario the rate region under a peak power constraint can be expressed as

$$\bar{\mathrm{R}}_{\mathrm{orth}} = \bigcup_{\substack{0 \le \tau \le 1 \\ 0 \le P_1 \le \bar{P},\, 0 \le P_2 \le \bar{P}}} \left(\tau R_1, (1-\tau) R_2' \right)$$



Generally, we have that $\bar{\mathrm{R}}_{\mathrm{orth}} \subseteq \bar{\mathrm{R}}$

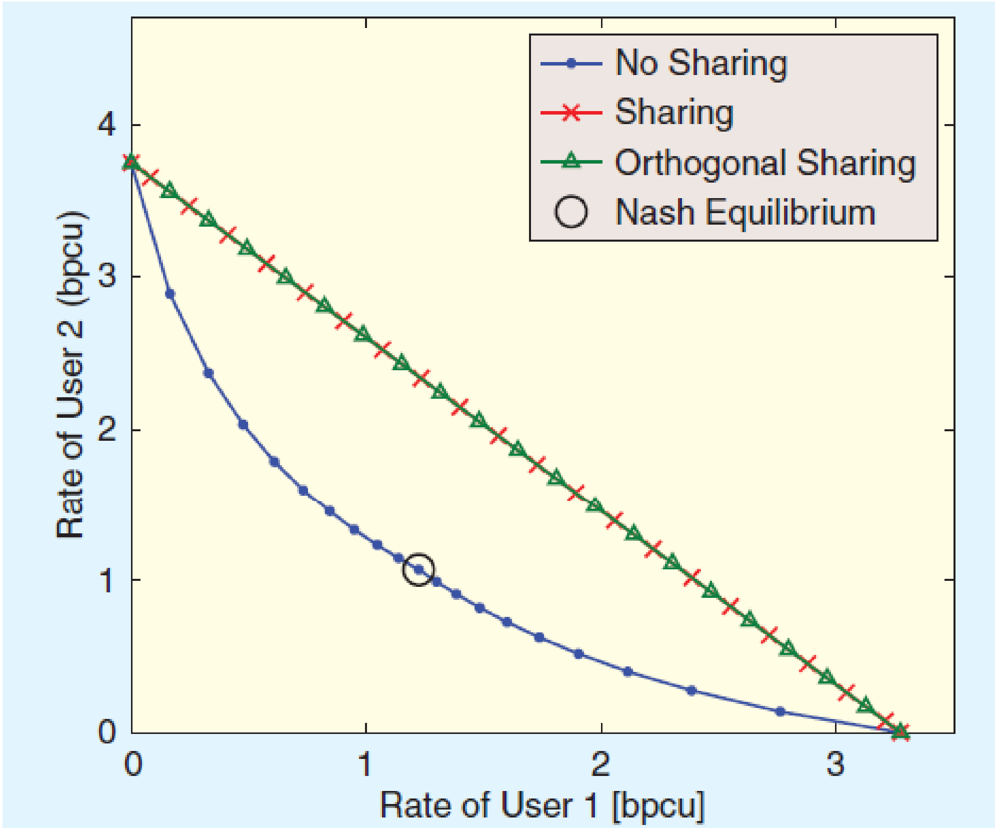


Weak interference between the systems

• The following two comments can be expressed for the last figure:

- R is a **convex region**, so that time-sharing cannot enlarge it.

- Orthogonal time-sharing shrinks the rate region!



Strong interference between the systems

• In the last case R is **nonconvex**, so that time-sharing enlarges the rate region.

• In this case, there is no loss induced by forcing the time-sharing to be orthogonal.

• The channel realizations in this example were chosen such that both $|h_{11}|^2$ and $|h_{22}|^2$ have the same values for the weak and the strong interference case. This is the reason for why the regions $\overline{\mathrm{R}}_{\mathrm{orth}}$ are the same in the two cases.

• The basic problem with the power game we are considering is that if the systems act **unilaterally** (**not cooperating**); then no system has any incentive to do time-sharing (stop transmitting for a period) nor to transmit with less than the maximal possible power.

Game theory: basic definitions and notions

• The game described in the previous example offers an example of **noncooperative game**, i.e. of a game in which the players strictly compete and cannot strike deals.

• On the contrary, in **cooperative games**, players can negotiate with one another and form joint strategies.

• If two players do not cooperate, then the only reasonable operating point will be at a so-called **Nash equilibrium**.

•A **(Nash) equilibrium** is an operating point where no player can improve his situation by **changing his strategy unilaterally**, under the **assumption that all the other players continue their current strategy**. In other words each player can only loose by deviating by himself from the equilibrium. The Nash equilibrium is thus, in a sense, a **stable operating point** for a system defined by a game.

Example 3 (continued) –

In the power game of Example 3, the strategy space consists of the powers P_1, P_2 and the time-sharing factor τ.

First scenario: only the parameters P_1 and P_2 are considered. The Nash equilibrium is the set of P_1, P_2 for which

$$R_1^{NE} = \log_2\left(1+\frac{P_1^{NE}\left|h_{11}\right|^2}{P_2^{NE}\left|h_{21}\right|^2+\sigma^2}\right) \geq \log_2\left(1+\frac{P_1\left|h_{11}\right|^2}{P_2^{NE}\left|h_{21}\right|^2+\sigma^2}\right)$$

for all P_1 with $P_1 \leq \overline{P}$ and

$$R_2^{NE} = \log_2\left(1+\frac{P_2^{NE}\left|h_{22}\right|^2}{P_1^{NE}\left|h_{12}\right|^2+\sigma^2}\right) \geq \log_2\left(1+\frac{P_2\left|h_{22}\right|^2}{P_1^{NE}\left|h_{12}\right|^2+\sigma^2}\right)$$

for all P_2 with $P_2 \leq \overline{P}$.

Game theory: basic definitions and notions

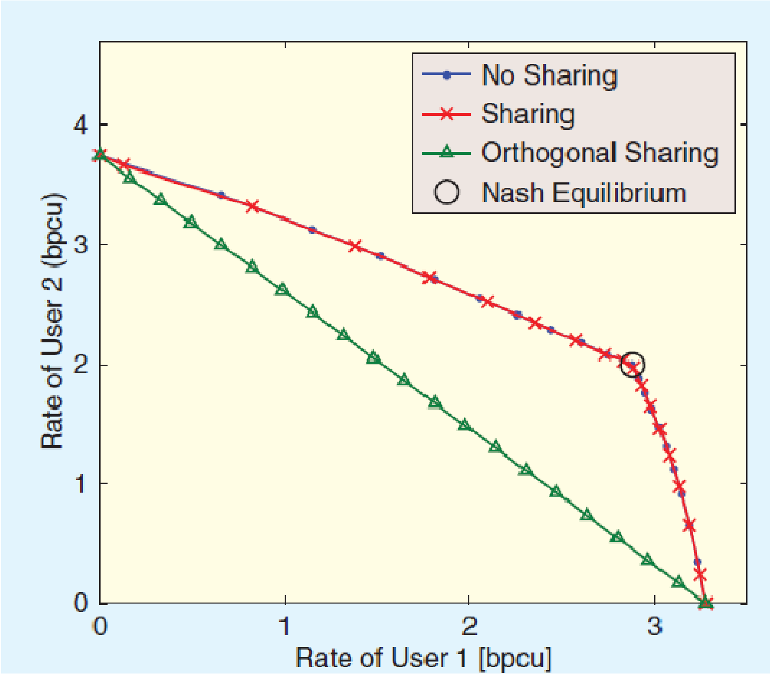
• In practice, we must also consider the time-sharing factor τ.

• In the power game, it turns out that there is a trivial Nash equilibrium that consists of transmitting with maximum power ($P_1 = P_2 = \bar{P}$) and not doing time-sharing*.

• To see that one should transmit continuously at the equilibrium, one can waterfill the available power over the noise and interference. If user two transmits with constant power over all time slots, then the waterfilling power allocation will give a constant power allocation for user one too. The Nash equilibria are shown in the following figures.

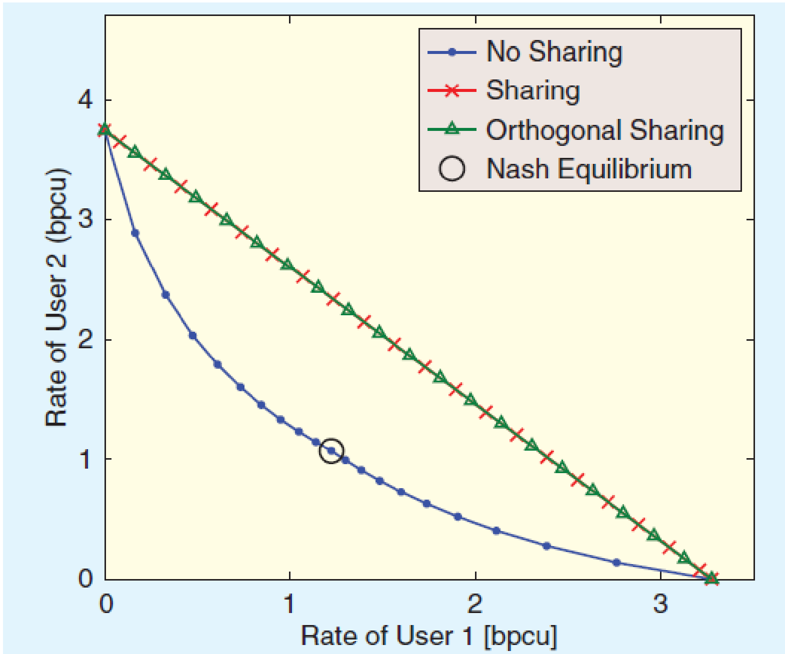
* R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicensed bands," *IEEE J. Select. Areas Commun.*, vol. 25, pp. 517–528, Apr. 2007.

Game theory: basic definitions and notions



Weak int.

Strong int.



• Very often, the **Nash equilibrium is a bad outcome** in the sense that the selfishness of players does not pay off.

• For example, in the power game with strong interference, any point on the single-user time-sharing line beats any point inside the region (especially the Nash equilibrium).

• However for weak interference, the Nash equilibrium is a good outcome in this example. Indeed, for the weak-interference case which the figure of the previous page refers to, the Nash equilibrium is sum-rate optimal.

Game theory: basic definitions and notions

• One characterization of the **efficiency** of the Nash equilibrium is the so-called **price of anarchy** (PoA).

• The PoA measures the cost that a system pays for operating without cooperation.

• It is defined as the ratio of the profit obtained at the **optimal** operating point, over the profit when functioning at the worst-case Nash equilibrium.

• The question arises here to the distinction of optimal operating points and what would be the “social good”.

• For this purpose, several global objective functions have been proposed, two of which are the utilitarian and the egalitarian solutions .

Game theory: basic definitions and notions

• The utilitarian solution could be the one of most interest to network operators, while the egalitarian solution could be perceived as "fairer".

• For instance, if we use the utilitarian **social welfare** function to express the PoA, this quantity in the IFC problem we considered is given by

$$\text{PoA}=\frac{\max\limits_{P_1,P_2}\left(R_1+R_2\right)}{\min\limits_{NE}\left(R_1^{NE}+R_2^{NE}\right)}$$

• The PoA is always greater than or equal to one. If PoA = 1, the NE achieves the utilitarian optimal solution. The PoA can be interpreted as follows: if e.g. PoA = 2, the optimal solution is twice as good as the selfish NE solution.

Game theory: basic definitions and notions

• The **social welfare** of a game (with *K* players) is defined as the sum of the utilities of all players

$$w = \sum_{i=1}^{K} u_i$$

• Social welfare, which corresponds to the average utility of the players (up to a scaling factor), is a well-known absolute measure of efficiency of a society, especially in economics.

• Is this quantity relevant in wireless communications? In theory, and more specifically in terms of ultimate performance limits of a network (Shannon theory), social welfare coincides with the **network sum-rate**.

Game theory: basic definitions and notions

• In contrast with many studies in economics, we have in communications, thanks to the Shannon theory, a fundamental limit for the social welfare. For example, if we have K terminals, each of them implementing a selfish PC algorithm to optimize his Shannon transmission rate and communicating with two BSs connected with each other, we know that the transmission rate of the equivalent virtual

$$K \times 2$$

multiple-input multiple-output (MIMO) system cannot be exceeded.

• In practice, it can be a good measure if the players undergo quite similar propagation conditions, in which case, the utilities after averaging (e.g., over fading gains) can be close. If the users experience substantially different propagation conditions the use of social welfare can be sometimes questionable and even leads to very unfair solutions.

Game theory: basic definitions and notions

• Social welfare has to be replaced, in some scenarios, with other measures of global network performance because: 1) as already mentioned, it can be unfair; 2) while it has a very nice physical interpretation when the users' utilities are chosen to be Shannon transmission rates, its meaning is much less clear in scenarios in which other utilities are considered (e.g., energy efficiency).

• Let us now go back to our analysis of Nash equilibria.

• Generally speaking, for a player, choosing a **pure strategy** consists of picking one element in his set of possible actions.

• It is worth pointing out that a **Nash equilibrium in pure strategies does not always exist**, as evidenced by the following Example.

Example 4

• Consider the two-player game defined as follows

$$A_i = \{0,1\} \quad u_i(a_1, a_2) = a_1 \oplus a_2 \oplus (i-1)$$

with $i = 1, 2$.

• Then, the first player's payoff is one when actions are different and zero otherwise, while the second player's payoff is one when the actions are identical and zero otherwise.

• Clearly, this game, also known as **matching pennies**, has no Nash equilibrium in pure strategies, since one of the players can always improve his situation changing his choice.

		Player #1	
		0	1
Player #2	0	**1, 0**	**0, 1**
	1	**0, 1**	**1, 0**

• Even when it exists, **the Nash equilibrium in pure strategies is not necessarily unique**, as shown in the following Example**.**

Example 5 - Two users are sharing an AWGN **multiple access channel** (i.e., joint user decoding at an access point)

The signal model is

$$y = x_1 + x_2 + n$$

where n is a Gaussian noise sample with variance σ^2. Each user has power P.

Game theory: basic definitions and notions

• It is well known* that the rate region of this multiple access channel is given by a pentagon defined by

$$R_1 \leq C^{\max} \qquad\qquad R_2 \leq C^{\max}$$

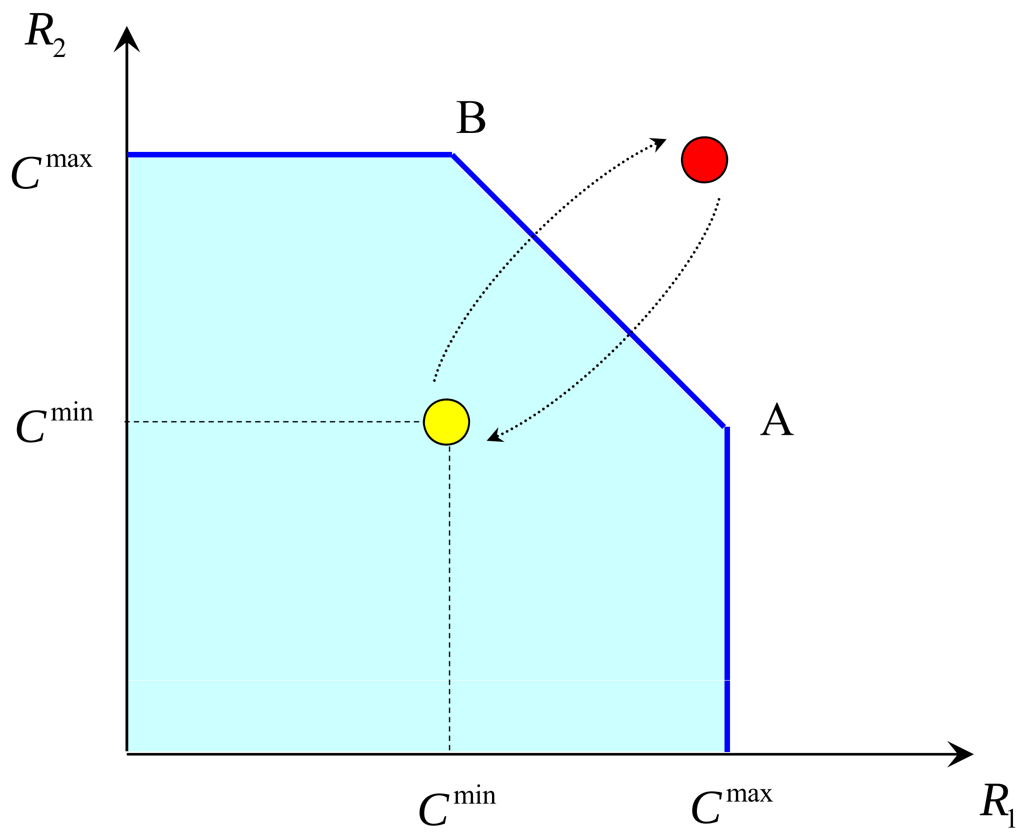
$$R_1 + R_2 \leq C_{1,2}$$

where

$$C^{\max} = \frac{1}{2}\log_2\left(1 + \frac{P}{\sigma^2}\right) \qquad\qquad C_{1,2} = \frac{1}{2}\log_2\left(1 + \frac{2P}{\sigma^2}\right)$$

* T. Cover and J. Thomas, *Elements of Information Theory*, Wiley Series in Telecommunications, New York: Wiley, 1991.

Game theory: basic definitions and notions



$$C^{\min} = \frac{1}{2}\log_2\left(1+\frac{P}{\sigma^2+P}\right)$$

is the rate achievable if its assumed that the other user's signal is interference.

• Any point on the line connecting the points A, B is achieved by **time sharing** between these two points. The *n*-th player can choose a strategy

$$0 \leq \alpha_n \leq 1$$

which is the **time sharing ratio** between coding at this rate at point A or B.

• The payoff in this game is given by

$$u_n(a_1, a_2) = \begin{cases} \alpha_n C^{\max} + (1-\alpha_n) C^{\min} \text{ if } \alpha_1 + \alpha_2 \leq 1 \\ 0 \text{ otherwise} \end{cases}$$

• The utility is 0 when

$$\alpha_1 + \alpha_2 > 1$$

since no reliable communication is possible (the rate pair achieved is outside the rate region).

• In this game, any valid strategy point such that

$$\alpha_1 + \alpha_2 = 1$$

is a **Nash equilibrium**. If the *n*-th user reduces his α_n obviously his rate is lower since he transmits a larger fraction of the time at the lower rate. If, on the other hand, he increases α_n , then

$$\alpha_1 + \alpha_2 > 1$$

and both players achieve zero.

Game theory: basic definitions and notions

- Hence, the considered game has an **infinite number** of Nash equilibria.

- To better understand this game, it is interesting to analyse the **best response** dynamics. The best response move is when a player attempts to maximize his utility against a given strategy vector. This is a well-established **means of distributively achieving the Nash equilibrium**.

- If, in a multiple access game, the players use the best response simultaneously, the first step would be to transmit at $C^{\max}$. Each player then receives zero utility and in the next step reduces his rate to $C^{\min}$, and vice versa.

- The **iteration never converges** and the utility of each player is given by $C^{\min}/2$

- This is worse than transmitting constantly at $C^{\min}$!

- Interestingly, in this case, the **sequential** best response leads to one of the points *A*, *B*, which are the (nonaxis) corners of the rate region (in this case the players select their strategies sequentially, not simultaneously).

- The moral of this story is that using the best response strategy should be done carefully.

- To overcome the problem of the lack of equilibrium in pure strategies, the notion of **mixed strategy** has been proposed.

• A mixed strategy π_n for player *n* is a probability distribution over his set of actions A_n .

• The interpretation of mixed strategies is that player *n* chooses his action randomly from A_n according to the distribution π_n .

• The **payoff** of player *n* in a game where mixed strategies are played is the **expected value of the utility**

$$u_n\left(\pi_1,\pi_2,\ldots,\pi_N\right)=E_{\pi_1\times\pi_2\times\ldots\times\pi_N}\left\{u_n\left(a_1,a_2,\ldots,a_N\right)\right\}$$

• In contrast with mixed equilibria, in **correlated equilibria** the lotteries used by the players can be correlated (by coordination signals), so that the joint probability distribution of the strategy of all the players cannot be factored.

Example 5 – continued

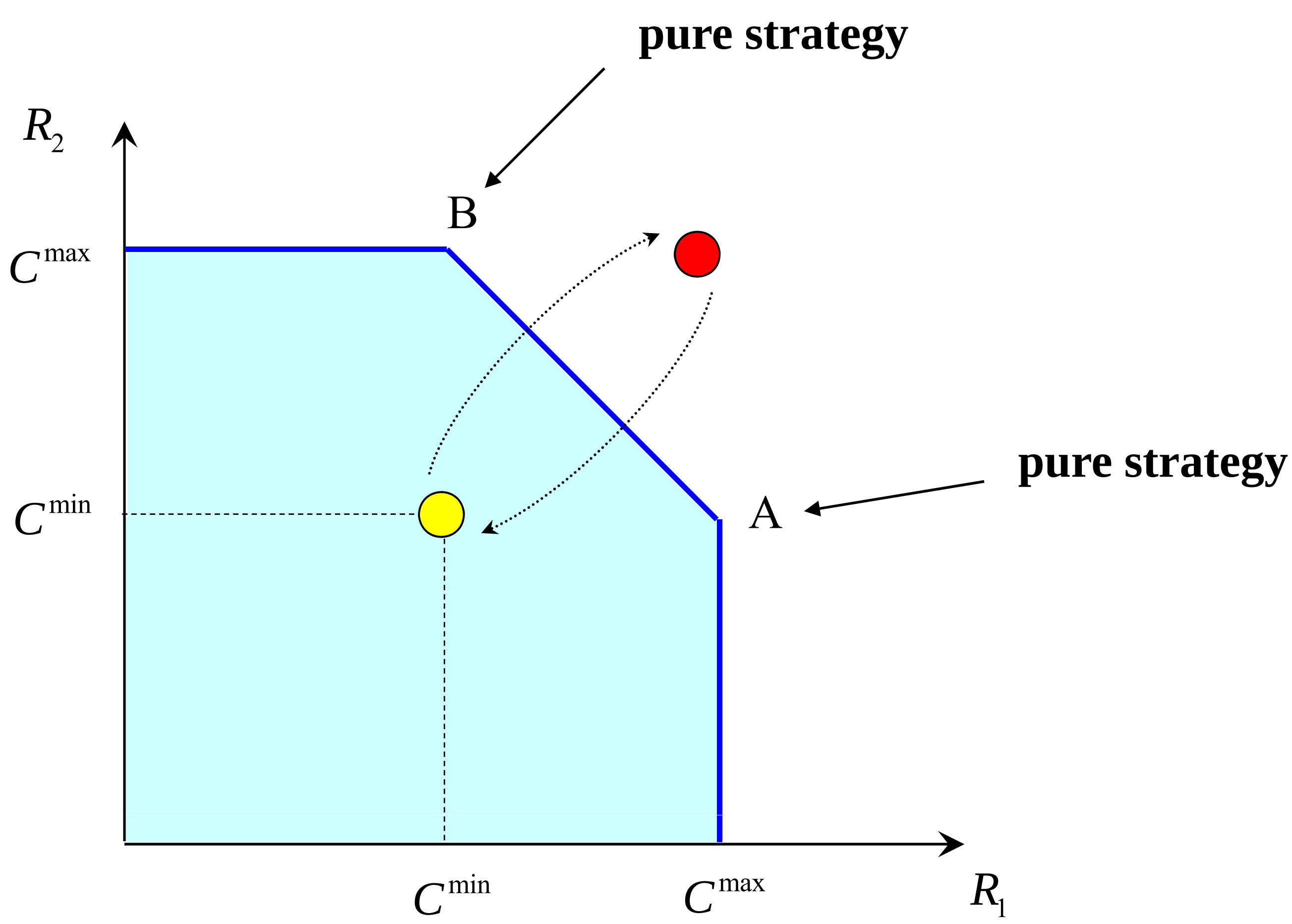
• To illustrate the notion of mixed strategy, we now extend the multiple access game to a **random multiple access game**, where the players can choose with probability p_n of working at rate $C_{\min}$ and $1-p_n$ at rate $C_{\max}$

• This replaces the synchronized TDMA strategy in the previous game with a **slotted random access protocol**.

• This formulation allows for **two pure strategies** corresponding to the corner points *A*, *B* and the mixed strategies amount to randomly choosing between these points.

• This game is a special case of the **chicken dilemma**, since for each user it is better to "chicken out" than to obtain zero rate when both players choose the tough strategies.

Game theory: basic definitions and notions



• Obviously, from the previous discussion,

$$\left(C_{\min}, C_{\max}\right) \quad \text{and} \qquad \left(C_{\max}, C_{\min}\right)$$

are Nash equilibria.

• Simple computation shows that there is a **unique Nash equilibrium in mixed strategies** corresponding to

$$p_1 = p_2 = C_{\min} / C_{\max}$$

• Interestingly, the rates achieved by this random access (mixed strategy) approach are exactly

$$\left(C_{\min}, C_{\min}\right)$$

i.e., the price paid for random access is that both players achieve their minimal rate, so simple *p*-persistent random access provides no gain for the multiple access channel. We can call this loss the **price of random access.**

Example 2 – continued

• In the Battle of the Sexes the (Rolling Stones, Rolling Stones) strategy pair, like the (Mozart, Mozart) pair, is a *Nash equilibrium* in the case of pure strategy.

• "The Battle of the Sexes" does have a Nash equilibrium in mixed strategies. In the mixed strategy Nash equilibrium, each player chooses his or her preferred concert with probability **2/3** and chooses the other concert with probability **1/3**. This equilibrium gives each player an expected payoff of **2/3.**

Game theory: basic definitions and notions

Eloise

		Rolling Stones	Mozart
Abelard	Rolling Stones	2, 1	0, 0
	Mozart	0, 0	1, 2

Nash equilibria (pure strategy)

• The last examples have evidenced that Nash equilibria **do not always entail the same payoffs**.

• For instance, the three equilibria we have identified in the last game offer three different payoffs to each of the players.

• The concept of **Pareto efficiency** can be used to compare different outcomes. An outcome is said to be **Pareto efficient** if it is impossible to increase the payoff of any player without decreasing the payoff of another player.

• In the last example, the pure strategy Nash equilibria are Pareto efficient; the mixed strategy equilibrium is not.

Game theory: basic definitions and notions

• The economist and Nobel Laureate John Nash showed that if each player in an n-player game has a finite number of pure strategies, then the game has a Nash equilibrium in pure or mixed strategies.

• Nash equilibria are often associated with "**rationality**". In other words, it would be irrational for players with complete knowledge of the game to choose any combination of strategies that does not constitute a Nash equilibrium.

• In the realm of economics, people often select strategies that are not rational. Fortunately, we need not worry about this issue, for we are interested in programmed agents that will always do what we tell them to do.

- **Existence of Nash equilibria**

  Equilibrium existence theorems are based on **topological properties of the strategy sets** of the players and **topological and geometrical properties of their utility**.

Example: Debreu-Fan-Glicksberg theorem (1952)

Let G be a static strategic noncooperative game. If for each n

1. A_n is a compact and convex set;

2. $u_n\left(a_1, a_2, ..., a_N\right)$ is a continuous function in the profile of strategies $\left(a_1, a_2, ..., a_N\right)$ and quasi-concave in a_n ;

Then the game G has at least one pure NE.

Game theory: basic definitions and notions

- A function $f(\mathbf{x})$ is **quasi-concave** on a convex set **S** if, for all $\alpha \in \Re$, the upper contour

$$\mathbf{U}_{\alpha} = \left\{ \mathbf{x} \in \mathbf{S} : f(\mathbf{x}) \geq \alpha \right\}$$

is convex.

- A special case of this theorem is when the utility functions are **concave** (**convex game**).

- The theorem by Rosen* for concave *K* person games can be seen as a corollary of above mentioned theorem.

* J. Rosen, "Existence and uniqueness of equilibrium points for concave *n* person games," *Econometrica*, vol. 33, no. 3, pp. 520–534, 1965.

Game theory: basic definitions and notions

- **Convex competitive games** are especially important in the context of spectrum management, since the basic Gaussian interference game forms a convex game.

- Existence theorems are not require to explicate the best responses (BRs) of the players; the BR of player *i* corresponds, by definition, to the set of strategies

$$BR_i\left(\mathbf{a}_{-i}\right)$$

maximizing the utility of user *i* when the rest of the world plays $\mathbf{a}_{-i}$:

$$BR_i\left(\mathbf{a}_{-i}\right) = \arg\max_{\tilde{a}_i} u_i\left(\tilde{a}_i, \mathbf{a}_{-i}\right)$$

- In general, $BR_i\left(\mathbf{a}_{-i}\right)$ can be a correspondence, but it is a function in many wireless games addressed in the current literature of communications.
- If the BRs can be explicated, the existence proof is equivalent to proving that the BRs have a non-empty intersection, which can be very simple in some scenarios.
- A natural question would be to ask whether the Debreu- Fan-Glicksberg theorem has a counterpart for **uniqueness**, that is, there exists a general uniqueness theorem for quasi-concave *K* player games. Unluckily, the answer is negative.
- However, there is a powerful tool for proving the uniqueness of a pure NE when the players' utilities are concave: this tool is the **uniqueness theorem** derived by Rosen [1]. This theorem states that if a certain condition, called **diagonally strict concavity** (DSC), is met, then uniqueness is guaranteed.

• If the BR of every player can be expressed, it is possible to analyze their properties and for some classes of functions (or correspondences) to characterize the number of intersection points between them. The number of intersection points corresponds to the number of equilibria.

• There are important scenarios where the NE is not unique (e.g., in routing games and in games where the choice of actions from different players is not independent). Some natural questions that arise concern the selection of an appropriate equilibrium:

- What can be done when one has to deal with a game having multiple equilibria?
- Are they some dominant equilibria?
- Are there some equilibria fairer and more stable than others?

Game theory: basic definitions and notions

• So far, we have mainly focused on ***static games with complete information***; in other words, the game is played in one shot, based on the fact that every player knows everything about the game.

• It is in this precise framework where existence and uniqueness of an NE has been discussed in the previous slides. Interestingly, the Nash equilibria predicted in such framework can be observed in others that are less restrictive in terms of information assumptions.

• These other frameworks include the situation where each player observes the actions played by the others, react to them by playing his BR, the others update their strategy accordingly, and so on. It turns out that these games can converge to an NE that would be obtained if the players knew the game completely and played it in one shot. In the presence of multiple equilibria, convergence to a specific NE will depend on the game starting point.

Game theory: basic definitions and notions

• For instance, the initial operating state of a network can determine the equilibrium state in decentralized networks having certain convergence properties.

• It is important to know that games with standard BRs have attractive convergence properties. For example, in ref. * it is shown how simple learning procedures, based on mild information assumptions, entail converge to the NE predicted in the associated game with complete information.

* P. S. Sastry, V. V. Phansalkar, and M. A. L. Thathachar, “Decentralized learning of Nash equilibria in multi-person stochastic games with incomplete information,” *IEEE Trans. Syst., Man, Cybern.*, vol. 24, no. 5, pp. 769–777, May 1994.

• More generally, if there is a certain **hierarchy** in the game, this hierarchy can be exploited by one or several players to enforce a given equilibrium.

• The desired equilibrium can be selected because of its efficiency. Therefore, equilibrium efficiency is also a way of selecting an equilibrium. **The fact that this equilibrium will effectively occur depends on whether there exists an entity capable of influencing the game**. For instance, in the scenario of ref. *, where two point-to-point communications compete with each other (interference channel), the network owner chooses the best location for the added relay to maximize the network sum-rate at the equilibrium.

* E. V. Belmega, B. Djeumou, and S. Lasaulce, “What happens when cognitive terminals compete for a relay node?” in *Proc. IEEE Int. Conf. Accoustics, Speech and Signal Processing* (*ICASSP*), Taipei, Taiwan, Apr. 2009, pp. 2609–2612.

Example 6

• The following Figure shows a two-player normal form of the famous game known as the *Prisoners' Dilemma*.

• Here a slightly modified version of the usual scenario is given. Two senators have been caught accepting bribes. **There is not sufficient evidence for a full conviction of both senators, so the Justice Department offers each of them a deal**.

• In practice, each is told that they may either confess and testify against the other (the ***defection*** **strategy**) or remain silent and suffer the potential consequences (the ***cooperation*** **strategy**).

• The resulting payoffs are shown in the following figure in the form of the resulting prison sentence; obviously, players in this game would prefer lower payoffs.

Game theory: basic definitions and notions

		Senator #1	
		Do not confess	Confess
Senator #2	Do not confess	**1 yr, 1 yr**	**10 yrs, 0 yrs**
	Confess	**0 yrs, 10 yrs**	**4 yrs, 4 yrs**

• The original version of the Prisoners' Dilemma is due to **Merrill Flood** and **Melvin Dresher**. They developed the game at the RAND Corporation while trying to model the interaction of the then Soviet Union and the United States in a nuclear standoff.

• Note that **this game has only one Nash equilibrium** (corresponding to (confess, confess)).

• The Prisoners' Dilemma becomes even more interesting if we choose to **repeat the game**.

• First consider the case in which the players are told that they will **repeat the game a fixed and finite number of times**. We assume that their sentences for the entire game are the sum of their sentences at each stage. Such a repeated game is said to have *a finite horizon.*

• You might imagine that the prisoners would want to cooperate with each other; if they didn't, their counterpart would punish them in the next stage.

• **Due to the finite horizon, this is not the case**. In the last game, there is no possibility of punishment in a subsequent game, so the **rational players defect**. They know this, so at the penultimate game they also defect, and so on back to the first game.

• Now suppose that the players do not know when the game is going to end. In this ***infinite horizon*** **game** there is always a possibility of punishment, and **one Nash equilibrium strategy profile is for each player to always cooperate** unless his or her opponent has defected in the past, in which case he or she always defects.

Game theory: basic definitions and notions

• When a specific game is analyzed as a one-shot game, users are **myopic**; their only concern is the current value of the utility function.

• By modeling a game as a **repeated game**, we create users who can consider the consequences of their actions. A user who "cheats" in the current time slot may be punished by other users in future time slots.

• A ***general strategy for a repeated game*** specifies the player's (user's) action for each possible game history.

• Implementing an arbitrary strategy is extremely difficult, though. The usual restriction, then, is that each player's strategy be implementable with a finite-state machine.

• Each state specifies the strategy that will be played. After each repetition of the constituent game, the outcome of the game determines the transition between states.

Game theory: basic definitions and notions

• In **cooperative games**, players (here: systems) are allowed to bargain and strike deals with one another.

• The theory for cooperative games splits into the cases of **transferable utility** and **nontransferable utility**. In the case of transferable utility, the players can pay one another side payments; with nontransferable utility, this is not allowed.

• A fundamental point we must understand is that a player can be **cooperative** and **rational** at the same time. That is, being cooperative does not mean the same thing as being altruistic. The point is that even if players are eventually interested in maximizing their own outcome, they may be willing to accept a bargaining solution that is found to be good enough for both.

• One way of modeling this behavior mathematically is by using Nobel laureate (economics) John Nash bargaining theory [21].

Example 7

• This classic example is meant to illustrate the basic issues involved in modeling bargaining situations.

• **Two men, one rich and one poor, meet a genie on the street. The genie offers them euro 100 to share, provided that they can agree on how to split the money.**

• What will be the outcome of this event? The question, while somewhat imaginary, captures the same fundamental behavioral issues encountered in game models for communication problems. Thus, if we can understand how to deal with this question, we will also have gained some insight into games concerning power allocation and beamforming.

- The **Nash bargaining theory** answers the euro 100 question by formulating a set of axioms and proving the existence of a unique “bargaining solution.”

- The Nash bargaining theory predicts to some extent what is likely to happen in practice if all parties act strictly rationally.

- An important point is that Nash bargaining has nothing to do with the Nash equilibrium, since the latter applies only to noncooperative games and there bargaining makes no sense.

- The Nash bargaining outcome is not necessarily “fair”, if we follow the definition of fairness adopted by most people; this is evidenced by the Nash solution to the euro 100 question…

Example 7 - continued

- Let us solve now the $100 question by using the Nash bargaining theorem.

- Assumptions:

1. the utility of money is logarithmic in the amount owned;

2. the rich man (R) is near infinitely rich ($x_R = 10^{10}$);

3. the poor man (P) owns only $x_P = 10$ in total.

- Let x be the amount R gets in the bargain. After bargaining the utility for R is

$$u_R^{\text{bargain}} = \log(10^{10} + x)$$

and the utility for P is

$$u_P^{\text{bargain}} = \log(10 + (100 - x))$$

- In this problem the so called **threat point** is given

$$\left(u_R^*, u_P^*\right) = \left(\log(x_R), \log(x_p)\right)$$

It refers to the case in which if no bargain occurs, so that both R and P will leave with exactly the initial amount they owned (in other words, the threat point is the outcome that is achieved if the players cannot agree on any bargaining outcome).

• The Nash bargaining theorem establishes that the bargaining solution corresponds to the solution of the following problem

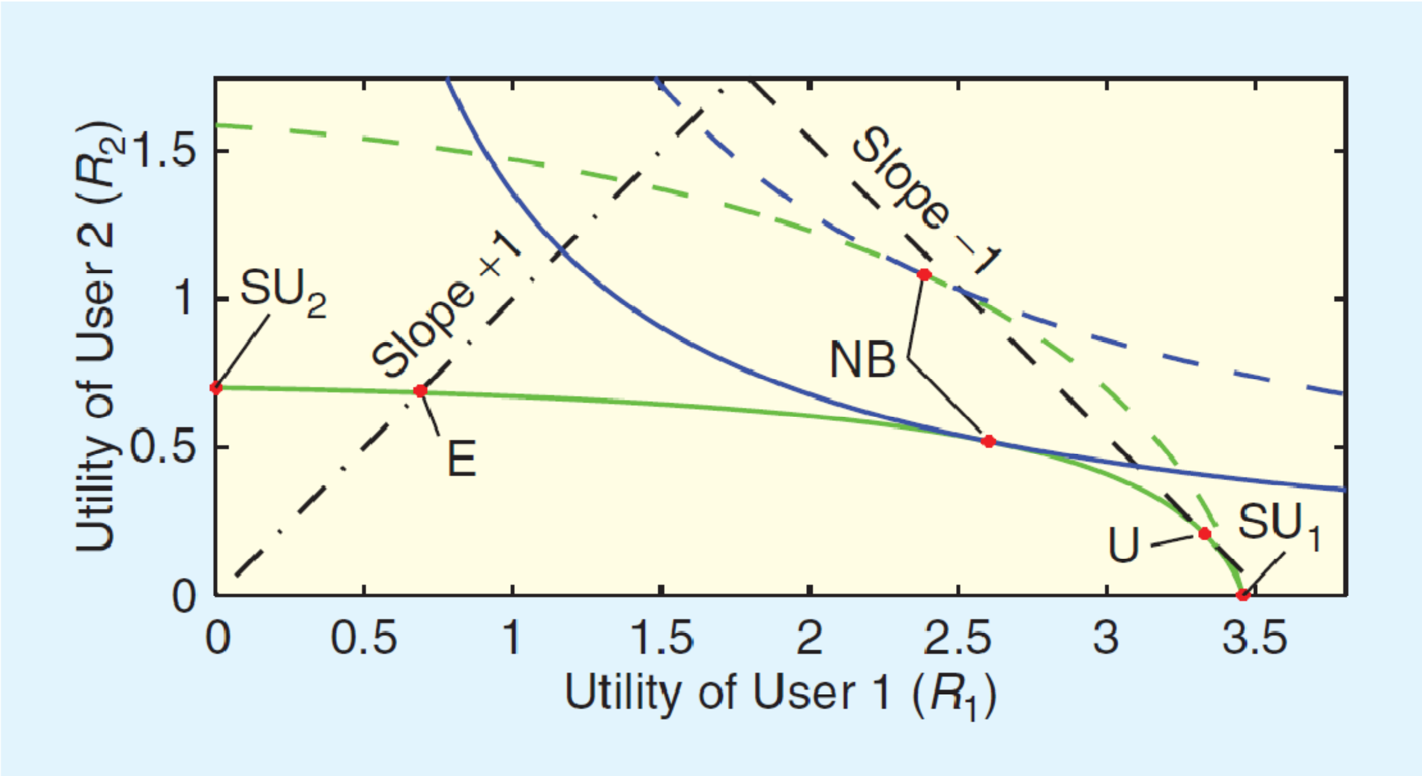
$$\max_{u_R, u_P} \left(u_R - u_R^*\right)\left(u_P - u_P^*\right)$$

with the constraint

$$x \in [0,100]$$

• The solution can be easily found graphically. It is the point where the Pareto boundary has a unique intersection with a hyperbola expressed as

$$\left(u_R - u_R^*\right)\left(u_P - u_P^*\right) = \text{constant}$$



NB = Nash bargaining

• The Nash bargaining solution of the euro 100-question is

$$x \cong 66$$

• Evidently, the bargaining outcome favors the rich man, who gets the most part of the money.

• For comparison, if instead P had initially owned only 10 cents (R has the same amount of money as before), then the Nash bargaining solution would be

$$x \cong 84$$

and the outcome would be even more unbalanced. The reason is that R has much more bargaining power. In fact, he can dictate a “my way or no way” outcome by threatening to walk away without a deal if he does not get a larger share of the money.

• Especially he knows that not being able to reach a deal will hurt P more than R so that P will be more willing to accept a bad deal than no deal at all.

• In the special cases when

$$x_P \to 0$$

the bargaining solution

$$x \to 100$$

and when

$$x_P \to x_R$$

the solution approaches

$$x \to 50$$

• Is the outcome of this example fair?

Game theory: basic definitions and notions

• This depends on how one defines **fairness**, which is by necessity a highly subjective notion.

• Bargaining theory does not aspire to model fairness in the sense that most humans interpret the term. Rather, it should be seen as a mathematical model for the fact that a stronger part in a (resource) conflict always has a larger power of negotiation and therefore will achieve a better outcome.

• When the performance of the network at the (considered) NE is found to be insufficient, the network or the game can be modified.

• There are many ways of doing this and we will just mention a few of them. What is important to have in mind is that the corresponding changes generally require allocating some resources (time-slots, band, . . .) for the nodes to exchange some information and implement the new strategies.

• **A possible way of improving the equilibrium efficiency is to transform the noncooperative game/network into a cooperative game/network**.

Game theory: basic definitions and notions

• Note that we distinguish between cooperative networks and cooperative games.

-In a **cooperative network** (say a cooperative MAC where transmitters exchange cooperation signals), the transmitters can still be selfish.

- In a **cooperative game**, some players help each other.

• For instance, in a large network it can be very useful to form smaller groups of players: this is the principle of **coalitional games**. If each member of a group or coalition has enough information, the coalition can even form a virtual antenna array, and the gain brought by cooperation has to be shared between the players of the group. In such games, we see that we can have both locally **cooperative networks** and a **noncooperative game** between the coalitions.

• Note that there are other forms of cooperation, sometimes more implicit. This is the case of **repeated games**.

• As it has been already stated, repeated games are a special case of dynamic games which consist in repeating at each step the same static game (the utilities result from averaging the static game utilities over time).

• In such games, certain agreements between players on a common plan can be implemented, and a punishment policy can also be implemented to identify and punish the deviators.

Game theory: basic definitions and notions

• The ways of improving the efficiency of the network equilibrium we have mentioned so far can be generally very demanding in terms of CSI at the transmitters and can possibly require to establish new physical links between some nodes.

• Since cooperation can be costly in terms of additional resources, a more reasonable solution can be to merely **coordinate the players**.

• This means adding a certain degree of coordination in a noncooperative game. Coordination between users can be stimulated, for instance, by using existing broadcasting signals like DVB or FM signals (case of public information), or by introducing dedicated signals sent by a BS (which can send both private and public information). The fact that all players of the game have access to certain (public and/or private) signals generally modify the players' behaviors. This knowledge can lead to a more efficient equilibrium (e.g., a new NE or even a correlated equilibrium)

• More generally, it can be shown that the set of achievable equilibria is enlarged by using common and private messages in the context of correlated equilibria.

• Finally, we will just mention two other usual techniques to improve the performance of a network at the equilibrium: implementing a pricing technique or introducing a certain degree of hierarchy in the game.

• What is the best technique to be used? The answer to this question depends on many factors. Among the dominant factors we have the **feasibility** of the technique, **predictability** of the effective network state, and the **performance** of the solution.

Game theory: basic definitions and notions

• **Feasibility** includes, for example, realistic information assumptions (CSI), complexity constraints at the terminals, and problems of measurability.

• **Unpredictability** can be the impossibility to prove the uniqueness of the equilibrium. For example, implementing pricing necessitates to modify the original utility functions and uniqueness (or predictability) can be lost after theses changes

• **Performance** can be a certain target in terms of quality of service.

A Game Theoretical Approach to the Management of Transmission Selection Scheme in Wireless Ad-Hoc Networks

Introduction

• In *wireless ad-hoc network* the connectivity between nodes can be achieved through multihop links; in such links multiple nodes can cooperate to form a *cluster* acting as a single stage for data relaying.

• Recently, substantial attention has been paid to the problem of cooperation in ad hoc networks consisting of *selfish nodes*, i.e. of nodes that aim at maximizing their own interest only; this problem has been tackled resorting to *game theory*.

• However, as far as we know, previous work in this area analyses only the problem of cooperation proneness of single nodes for data relaying.

• On the contrary, in the following, we provide a novel solution to the problem of both cooperation and coordination in a relay stage. Our solution is represented by a cooperative transmission strategy, functionally equivalent to a *transmission selection scheme*, but managed in a fully distributed fashion.

Introduction

• The proposed strategy is characterized by the following relevant features:

a) It *maximizes the individual utility of network nodes*, so that each node can earn *credits* with the minimum use of its radio resources;

b) it achieves high *efficiency* in the access to a shared medium;

c) it outperforms standard cooperative strategies based on transmission selection, even in terms of mean achievable throughput on a source-to-destination link;

d) it is characterized by autonomous observations and choices made by each node on the basis of its own profit, so that it works even in the presence of *selfish nodes*.

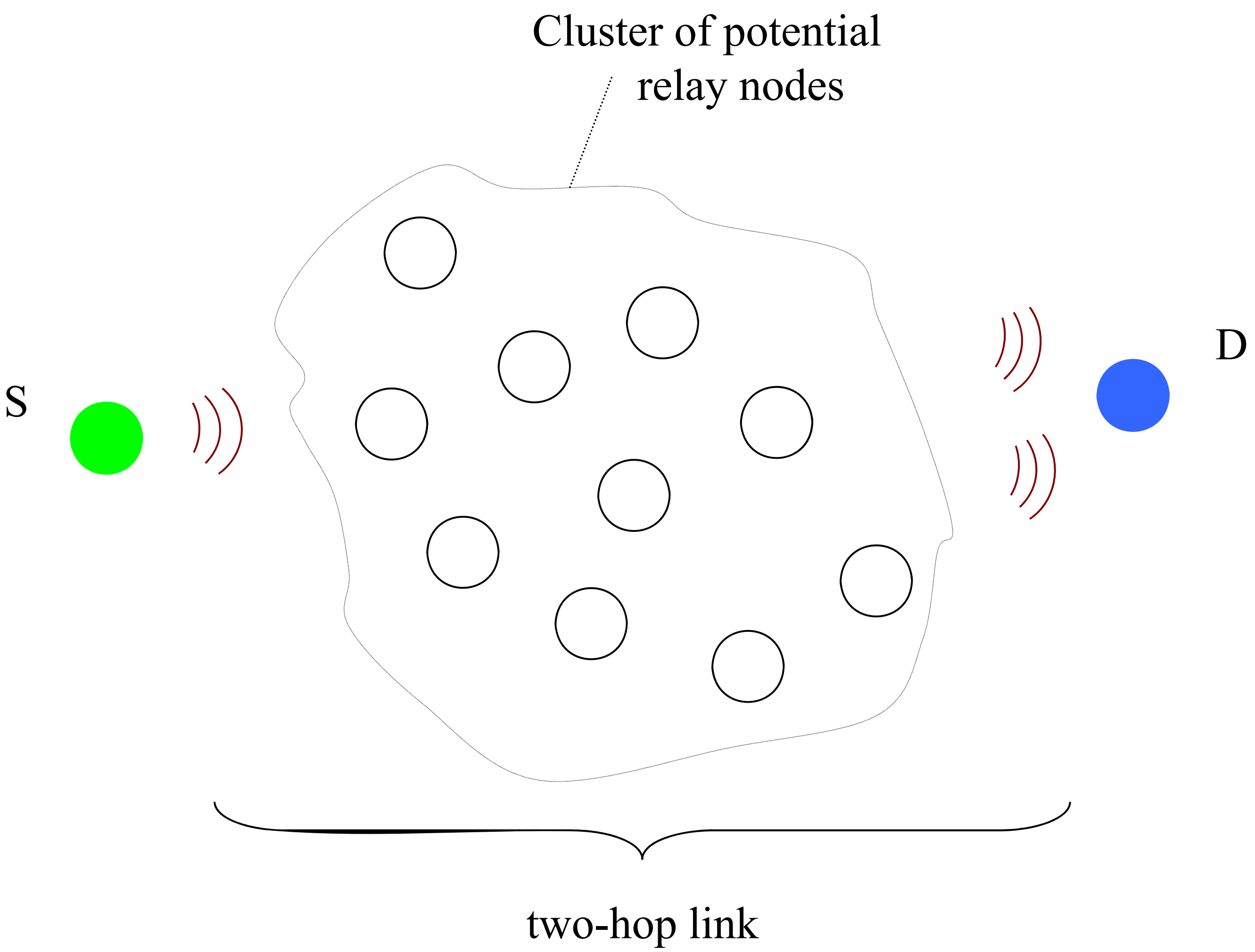
Network model

Assumptions:

1. A *double hop link*, i.e. a simple relay network, is analysed in the following;
2. A *source* (S) node needs to send a certain number of data packets to a *destination* (D) through a set of hierarchically equivalent and *rational* potential relay nodes, fully aware of their roles, each endowed with a *single antenna* and operating in a *decode and forward* fashion;
3. The source node does not reveal to the potential relays the total number of packets to be forwarded;
4. Each node is expected to behave in a selfish fashion, so that its intrinsic goal is to carry out its own data transmissions only, limiting its power consumption as much as possible.

Despite the last assumption, **each node can contribute to packet relaying**, since, as it will become clearer later, this results in earning credits exploitable for future communications.

Network model



Network model

- The solution developed below relies on a **simple economic model**; this establishes that the provision of a service, i.e., of packet relaying in this case, is rewarded with an economic counterpart, represented by a certain **amount of credits**.

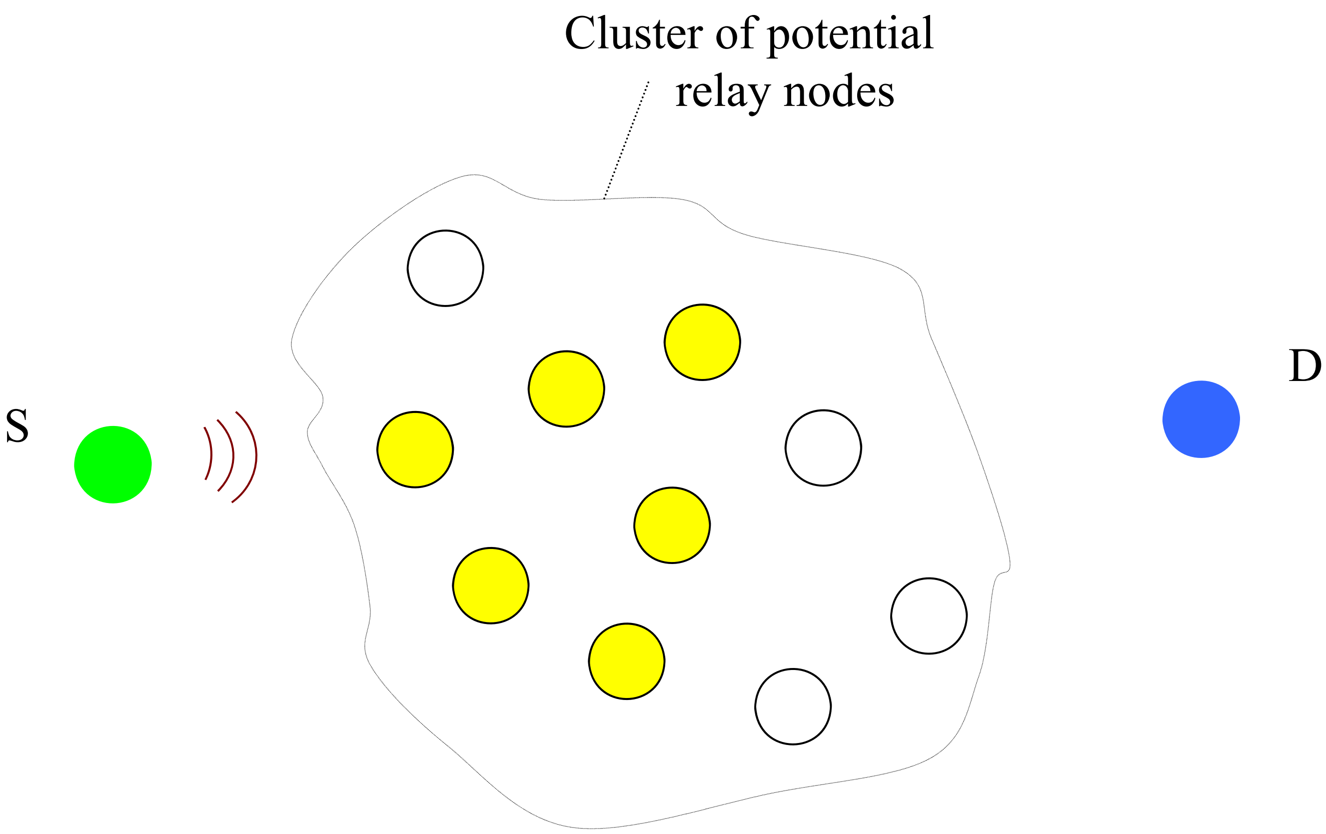
- The introduction of this policy for stimulating node cooperation justifies the need of broadening the considered system model from a simple relay network to a more generic ad hoc wireless network.

- In fact, if each node can act as both a relay and a source of information, it is really interested in earning the credits needed for its future data communications, consuming, at the same time, as few radio resources as possible.

Network model

• In the following, we focus on the data communication phase and, more specifically, on the S-to-D transmission of a **single packet**; this event consists of the steps described in the following slides.

Network model

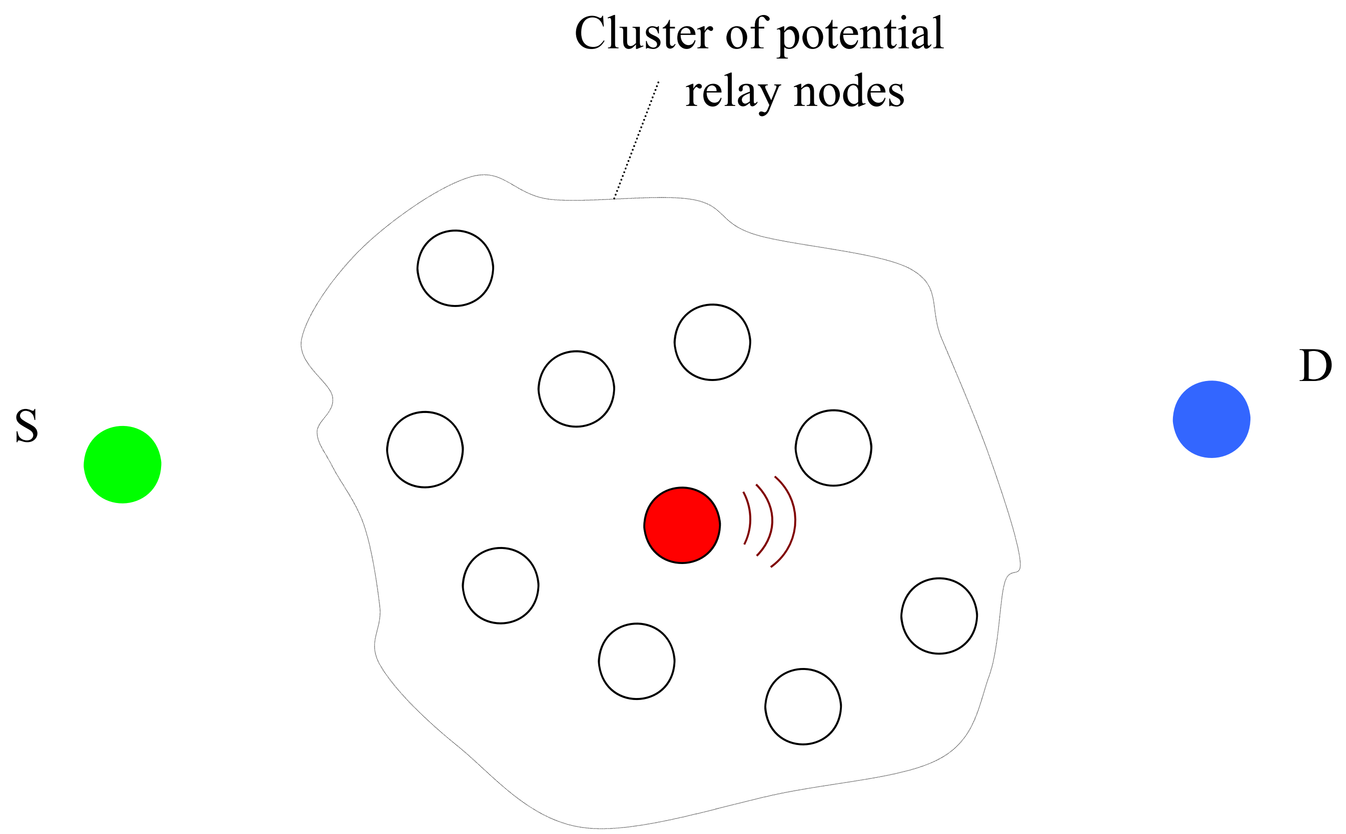


STEP #1- S sends a data packet to a potential relay set; such a packet, being broadcasted over the radio channel, is received by a set of nodes with a non zero probability.

Network model

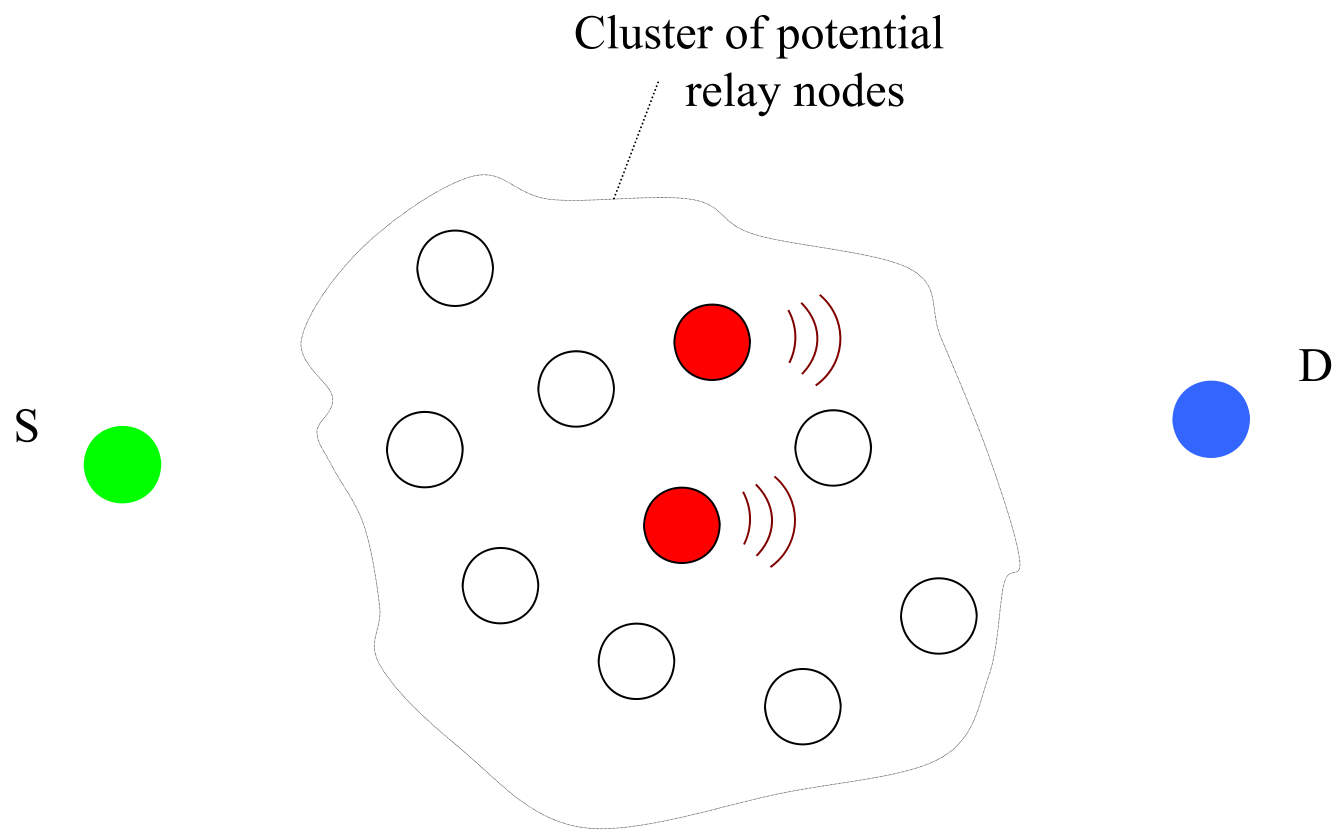
STEP #2 -Each node able to properly decode the packet represents a potential relay towards D within a virtual MISO link. It is assumed that the packets sent by S contain a known preamble which can be exploited by all the potential relays to achieve a rough synchronization only for their transmission. Then, considering the general case of unsynchronized and uncoded transmission, one of the events described in the followingh slides occurs.

Network model



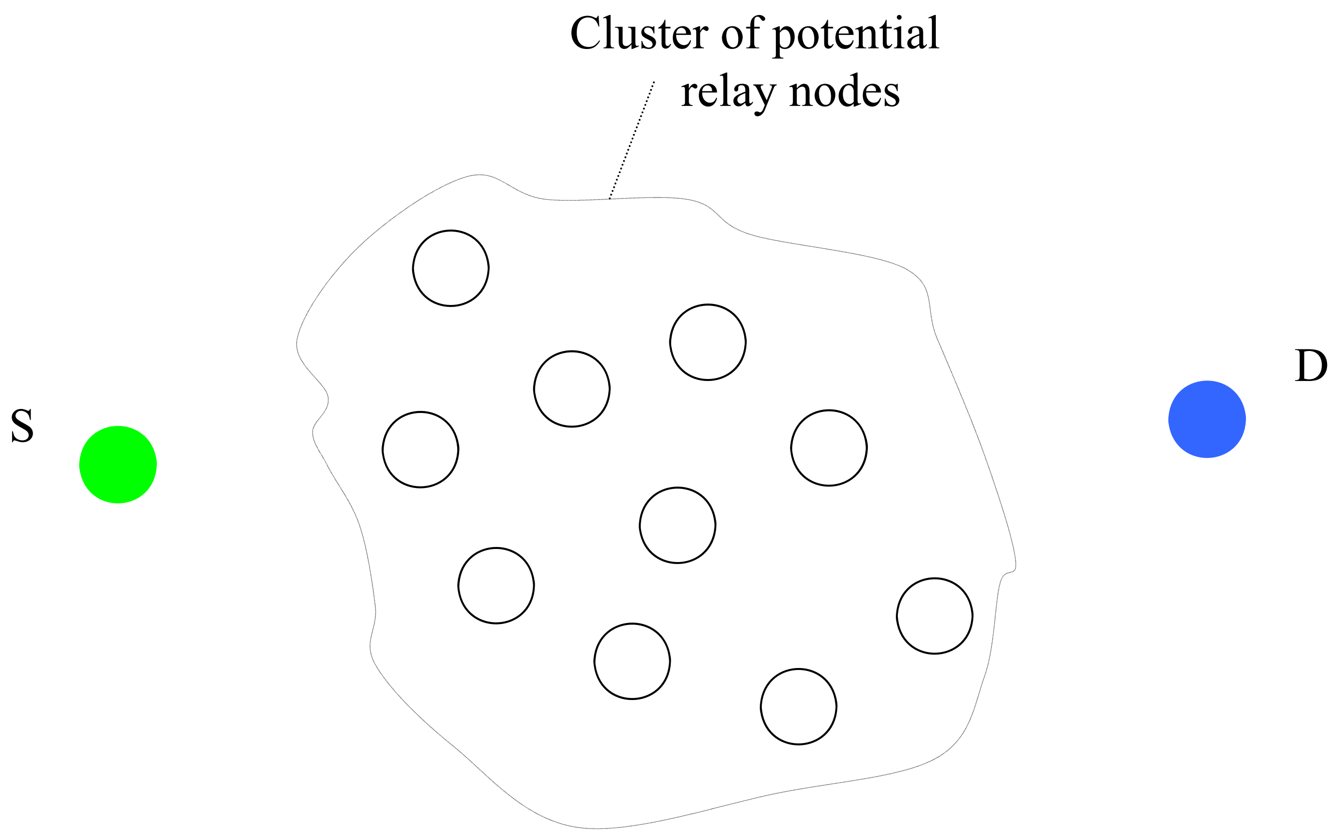
2.A - A single relay node forwards the packet to D. In this case, this node spends a fraction of its energy but, at the same time, earns a given amount of credits.

Network model



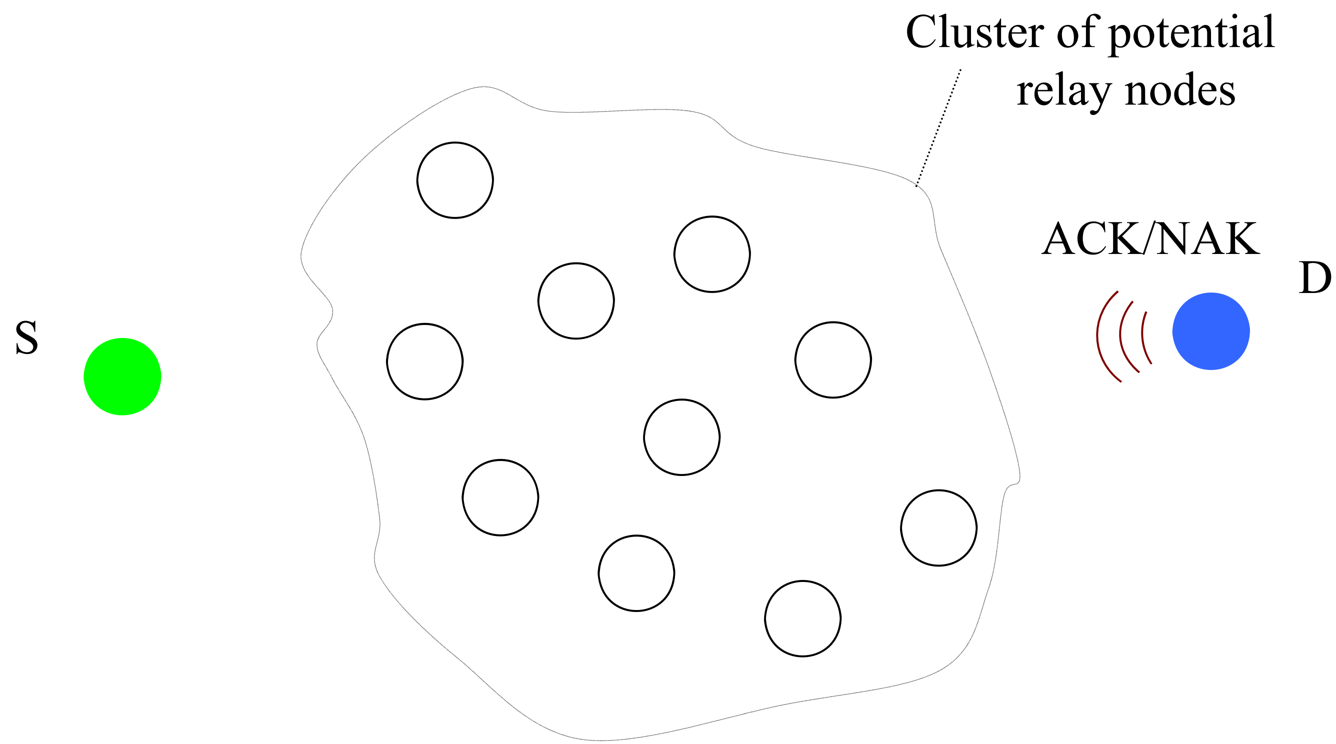
2.B – Multiple nodes transmit the same packet to D; this results in a **collision** and, consequently, in a waste of energy;

Network model



2.C - No node forwards the packet to D, so that such a packet is queued and transmitted later.

Network model



STEP #3 - D announces the outcome of the last transmission attempt broadcasting a single bit ACK/NAK feedback.
This feedback is supposed to be always correctly received by all the potential relay nodes which exploit it to estimate the quality of their channels towards D.

Network model

- Note that the cooperative transmission scheme resulting from the interaction of cluster nodes can be interpreted as a form of **transmit selection diversity**.

- Finally, it is worth mentioning that the following technical issues are not addressed in the following:

a) how credits are stored in each node;
b) how a consistent view of the credits in the ad-hoc network is kept;
c) how credit transactions are managed in a distributedfashion.

- Note, however, that these problems are common to most of the available solutions exploiting both virtual currency or reputation based techniques for stimulating node cooperation; for this reason, a distributed control scheme can be adopted.

Game model

• To ease the derivation of our strategy, the time axis is divided in slots (the slot length is equal to the duration of a data packet transmitted by a network node).

• **We focus now on a time instant in which the *n*-th relay node of the network needs to decide whether forwarding a data packet or not**.

• This transmission dilemma can be modeled as a ***multiplayer game*** in which, in principle, the set of players consists of the nodes belonging to the given relay cluster and the action set of each player is made of two distinct options (i.e., transmitting or remaining silent). Actually, in this game each player is interested only in adopting the strategy which can minimize the probability of collision for its transmissions, independently of the identity and of the number of the other nodes which can produce them.

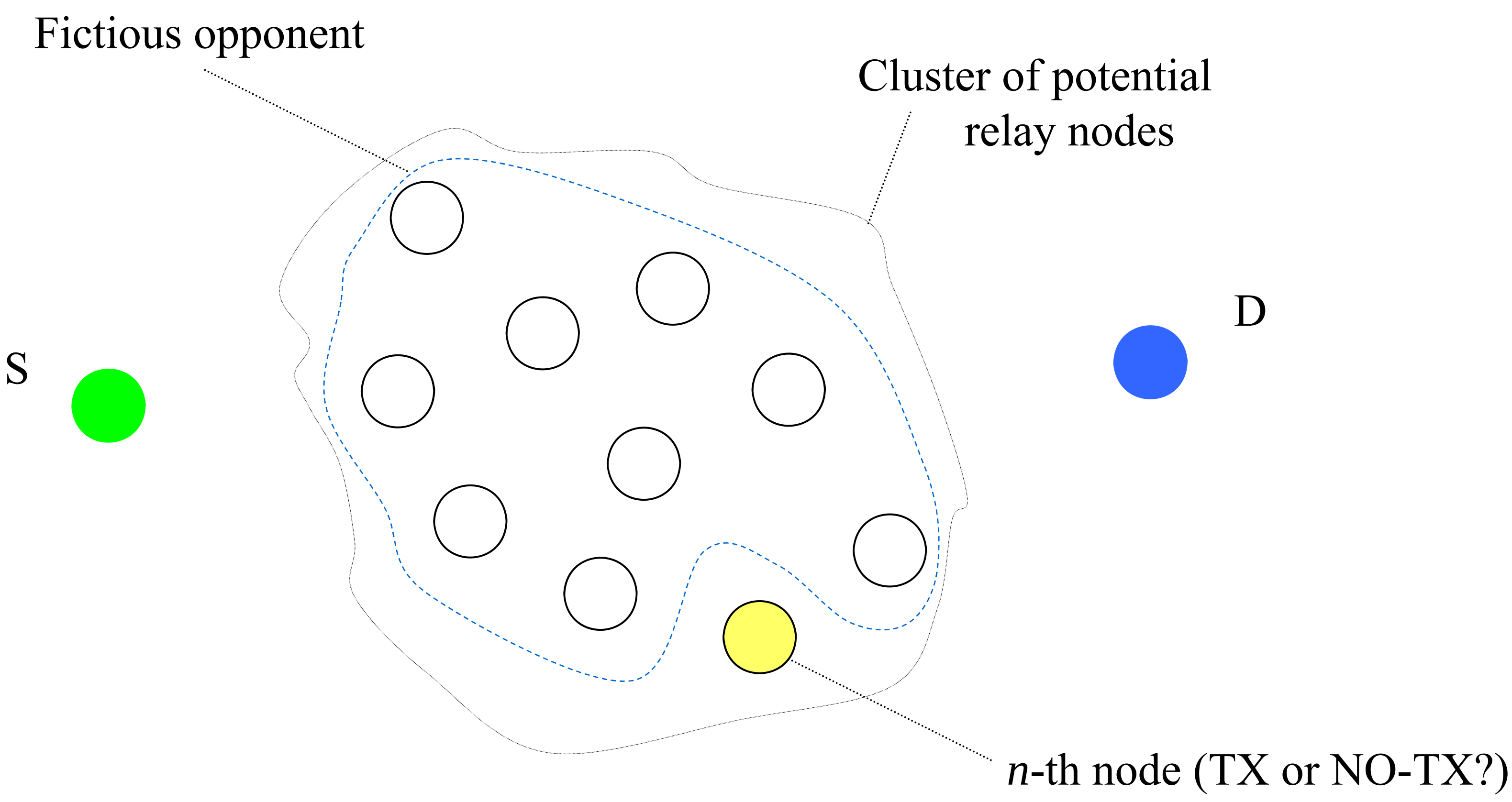
Game model

• For this reason, the transmission dilemma can be interpreted as a "challenge" between the considered node and the remaining nodes of its relay cluster, so that the original multiplayer game can be simplified, in the eyes of each node, into a ***fictitious two players game***.

• In this model, the considered node plays against a **single fictitious opponent**; in practice, the action of the opponent sums up the actions of the other nodes of the cluster, i.e. the opponent transmits when at least one node of the cluster decides to transmit, otherwise it remains silent.

• It is worth nothing that, since each node is able to acquire information about the behavior of its opponents only from a generic ACK/NAK feedback sent by the destination node, this model is in agreement with the scenario seen by the node itself.

Game model



Fictious opponent
Cluster of potential relay nodes
S
D
n-th node (TX or NO-TX?)

Game model

		n-th node: TX	*n*-th node: NO-TX
Fictious opponent	TX	$c_{fo,n}$, c_n	$a_{fo,n}$, 0
Fictious opponent	NO-TX	0 , a_n	0 , 0

- It is reasonable to assume that:

  $a_n > c_n$, since the *n*-th node is expected to deem a correct transmission of data packet more relevant than a collision;
- $c_n < 0$, since no reward is expected in the case of packet collision.

Game model

• Our game can be interpreted as a *chicken game*, in which the playing (transmitting) node wins when all the other nodes give up (stay silent).

• Both the payoffs $c_{fo,n}$ and $a_{fo,n}$ are unknown to the n-th node, since distinct nodes never share information. Therefore the game we are analysing belongs to the class of **games of incomplete information**.

Payoff a_n

• The payoff a_n expresses the additional benefit acquired by the *n*-th node when a packet is correctly transmitted

• In the solution we propose this payoff is expressed as a function of the **consumed resource** and of the **amount of earnable credits**.

• In practice, in our model the **amount of credits** earned by the *n*-th node in a given end-to-end communication is proportional to the overall number of packets that have been correctly forwarded by that node.

• However, in order to allow S to define a priori the overall amount of credits to be assigned to the complete relay stage, the number of packet transmissions accomplished by the *n*-th node is normalised with respect to the number of packet transmissions successfully carried out by the cluster it belongs to.

Game model

- **At the end of the t-th time slot** the **number of credits** earned by the *n*-th cooperating node in its transmission over a specific link is equal to

$$P(n,t) = B\frac{N_{tx}(n,t)}{\sum_{i \in C_n} N_{tx}(i,t)}$$

number of packets that have been properly forwarded by the *n*-th cooperating node until the *t*-th slot

overall number of packets sent by the whole relay set C_n, which the *n*-th node belongs to, over the same time interval.

overall amount of credits made available by S to reward the potential relay set for its efforts spent on the whole S to D link

Game model

• The rule expressed by the last expression

- allows the potential relays to assess in any slot their current contribution to the link and, consequently, the additional benefit coming from a packet transmission in the next slot.

- decouples the credits spent by S from, on the one hand, the number of packets that have to be transmitted and, on the other hand, the transmission scheme adopted by the relay set.

Game model

• The **energy consumed** by the *n*-th node **until the t-th slot** is given by

$$E(n,t) = \sum_{k=0}^{t} E_k(n)$$

energy spent by the *n*-th node over the *k*-th slot

Game model

• Given the quantities

$$P(n,t) \quad \text{and} \quad E(n,t)$$

the benefit deriving from the transmissions made until the current slot is evaluated as

$$f\left(P(n,t)\right)-g\left(E(n,t)\right)$$

• Here $f(\cdot)$ and $g(\cdot)$ are monotonic increasing functions having the specific purpose of making the contributions coming from these two quantities homogeneous and characterized by similar ranges (so that they play comparable roles in the evaluation of payoffs).

Game model

- If **one more packet** will be **successfully** forwarded by the n-th node in the $(t+d)$-th slot, both

$$N_{tx}\left(n,t\right) \quad \text{and} \quad \sum_{i\in C_n} N_{tx}\left(i,t\right)$$

increases by one, so that the amount of earnable credits becomes

$$P\left(n,t+d\right)=B\frac{N_{tx}\left(n,t\right)+1}{1+\sum_{i\in C_n} N_{tx}\left(i,t\right)}$$

- Moreover the energy spent $E\left(n,t\right)$ increases by $E_{t+d}\left(n\right)$, representing an estimate of the energy needed for the next transmission and evaluated from the knowledge of the channel attenuation in the previous slot.

Game model

- Then, a_n can be defined as

$$a_n \triangleq \left[f\left(P\left(n,t+d\right)\right) - g\left(E\left(n,t\right) + E_{t+d}\left(n\right)\right)\right] \\ -\left[f\left(P\left(n,t\right)\right) - g\left(E\left(n,t\right)\right)\right]$$

expressing the additional benefit acquired by the *n*-th relay node for the transmission of a new packet in the (t+1)-th slot.

Game model

Payoff c_n

• The payoff c_n assigned to the n-th node in case of packet collision can be evaluated resorting to the approach just described for a_n .

• If a collision occurs, the amount of packets which have been usefully forwarded by the *n*-th node and by all the other nodes of its cluster remains unchanged.

• Therefore, the payoff c_n can be expressed as

$$c_n \triangleq -g\left(E\left(n,t\right)+E_{t+d}\left(n\right)\right)+g\left(E\left(n,t\right)\right)$$

Game model

Risk affinity

The payoffs defined above do not take into account the willingness of the *n*-th node to spend its residual resources for data transmission. To account for this, the expressions of the payoffs a_n and c_n can be modified as

$$a_n \triangleq \left[w_{tx}(n)\, f\left(P(n,t+d)\right) - w_{en}(n)\, g\left(E(n,t) + E_{t+d}(n)\right)\right]$$
$$-\left[w_{tx}(n)\, f\left(P(n,t)\right) - w_{en}(n)\, g\left(E(n,t)\right)\right]$$

$$c_n \triangleq w_{en}(n)\left[g\left(E(n,t)\right) - g\left(E(n,t) + E_{t+d}(n)\right)\right]$$

respectively, where the weight $w_{tx}(n)$ ($w_{en}(n)$) measures the willingness of the n-th node to cooperate (to save its energy).

Game model

• Note that, since the functions $f(\cdot)$ and $g(\cdot)$ are generic, the real influence of these weights on the payoffs depends on their ratio, i.e. on the parameter

$$K_n \triangleq \frac{w_{tx}(n)}{w_{en}(n)}$$

which can be interpreted as the ***risk affinity*** for the *n*-th node.

•A large risk affinity pushes the *n*-th node to cooperate with the aim of earning as many credits as possible, in order to be able to support heavy traffic in the near future. A small risk affinity, instead, can be interpreted as an appreciable energy avidity, which pushes the terminal to cooperate scarcely.

•In the following we will always refer to the modified payoffs $\hat{a}_n$ and $\hat{c}_n$, which will be denoted a_n and c_n , respectively, to simplify the notation.

Opponent strategy

• In our model the payoffs $a_{fo,n}$ and $c_{fo,n}$, that can be acquired by the fictitious opponent, are unknown to the n-th node.

• However, this lack of information can be made up for exploiting the **repetitiveness of the game**. In particular, the transmission probability of the *fictitious opponent*

$$\Pr\left\{TX_{fo}\right\}$$

can be estimated as the ratio of the number $N\left(TX_{fo}\right)$ of transmission attempts (regardless if they have been successful or have produced a collision) carried out by the opponent relay cluster to the total number of transmission attempts, i.e. as

$$\Pr_{est}\left\{TX_{fo}\right\} = \frac{N\left(TX_{fo}\right)}{N\left(TX_{fo}\right) + N\left(no\_TX_{fo}\right)}$$

Game model

• Note that the knowledge of the slot period allows each node to count both the number of transmission attempts and the number $N\left(no\_TX_{fo}\right)$ of slots during which the relay cluster has remained silent.

• In the derivation of the strategy played by the *n*-th node it is important to keep in mind that the considered communication scenario cannot be deemed static if the wireless channel is affected by *time selectivity*.

• To overcome this problem, the probability $\Pr\left\{TX_{fo}\right\}$ can be estimated considering only the last N_{moves} (and not the entire history of the link).

- This means that, in the *k*-th slot,

$$N\left(TX_{fo}\right)\Big|_{k} \triangleq \sum_{l=1}^{N_{moves}} w(l)\,\delta_{TX}\left(k-l\right)$$

is used in place of $N\left(TX_{fo}\right)$ in

$$\Pr_{est}\left\{TX_{fo}\right\} = \frac{N\left(TX_{fo}\right)}{N\left(TX_{fo}\right)+N\left(no\_TX_{fo}\right)}$$

- Here the sequence $\delta_{TX}\left(l\right)$ is equal to unity if a packet transmission occurred in the *l*-th slot and to zero otherwise, and

$$w(l) \triangleq \frac{N_{moves}-l+1}{N_{moves}}$$

is a weight assigned to the move carried out *l* slots earlier.

Solving the game

• The game we are analysing is characterized by **3 Nash equilibria**. Two of them are pure equilibria and are trivially identified by the strategies

(TX, no TX) and (no TX, TX).

• Obviously, these two equilibria are useless for our application since do not result in a practically exploitable policy for the network nodes.

• The third equilibrium point, corresponding to a ***mixed strategy***, can be derived as explained below.

• If $\mathrm{P}_{fo}(TX)$ and $\mathrm{P}_{fo}(no\_TX)$ denote the actual probabilities with which fictitious opponent of the *n*-th node transmits and remains silent, respectively, the average payoff for the *n*-th node is given by

Solving the game

$$E_n\left(TX\right) = c_n\,\mathrm{P}_{fo}(TX) + a_n\,\mathrm{P}_{fo}(no\_TX))$$
$$= c_n\,\mathrm{P}_{fo}(TX) + a_n\left(1 - \mathrm{P}_{fo}(TX)\right)$$

if it decides to transmit, and by

$$E_n\left(no\_TX\right) = 0$$

in the opposite case.

• The mixed equilibrium point can be derived from the equality

$$E_n\left(no\_TX\right) = E_n\left(TX\right)$$

• It is easily inferred that the probability with which the opponent relay cluster transmits at the mixed equilibrium point is

Solving the game

$$\hat{\mathrm{P}}_{fo}(TX) = \frac{a_n}{a_n - c_n}$$

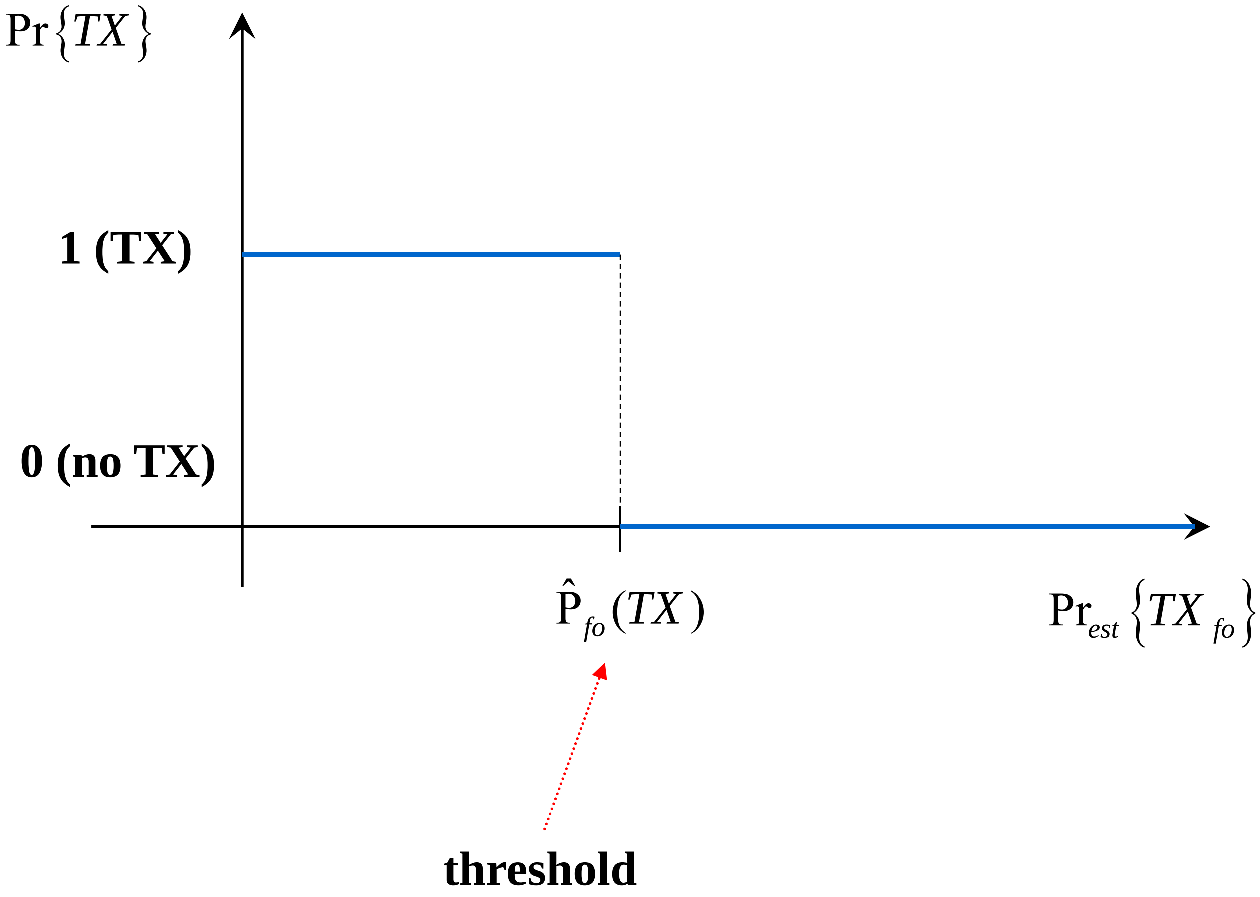
• The *best response* (BR) of the n-th node to the fictitious opponent actions is

$$\Pr_{est}\left\{TX_{fo}\right\} \le \hat{\mathrm{P}}_{fo}(TX) \quad \Longrightarrow \quad \textbf{TX}$$

$$\Pr_{est}\left\{TX_{fo}\right\} > \hat{\mathrm{P}}_{fo}(TX) \quad \Longrightarrow \quad \textbf{No TX}$$

• Note that the probabilities appearing in this strategy are evaluated by the *n*-th node on the basis of its information only is used.

Solving the game



Solving the game

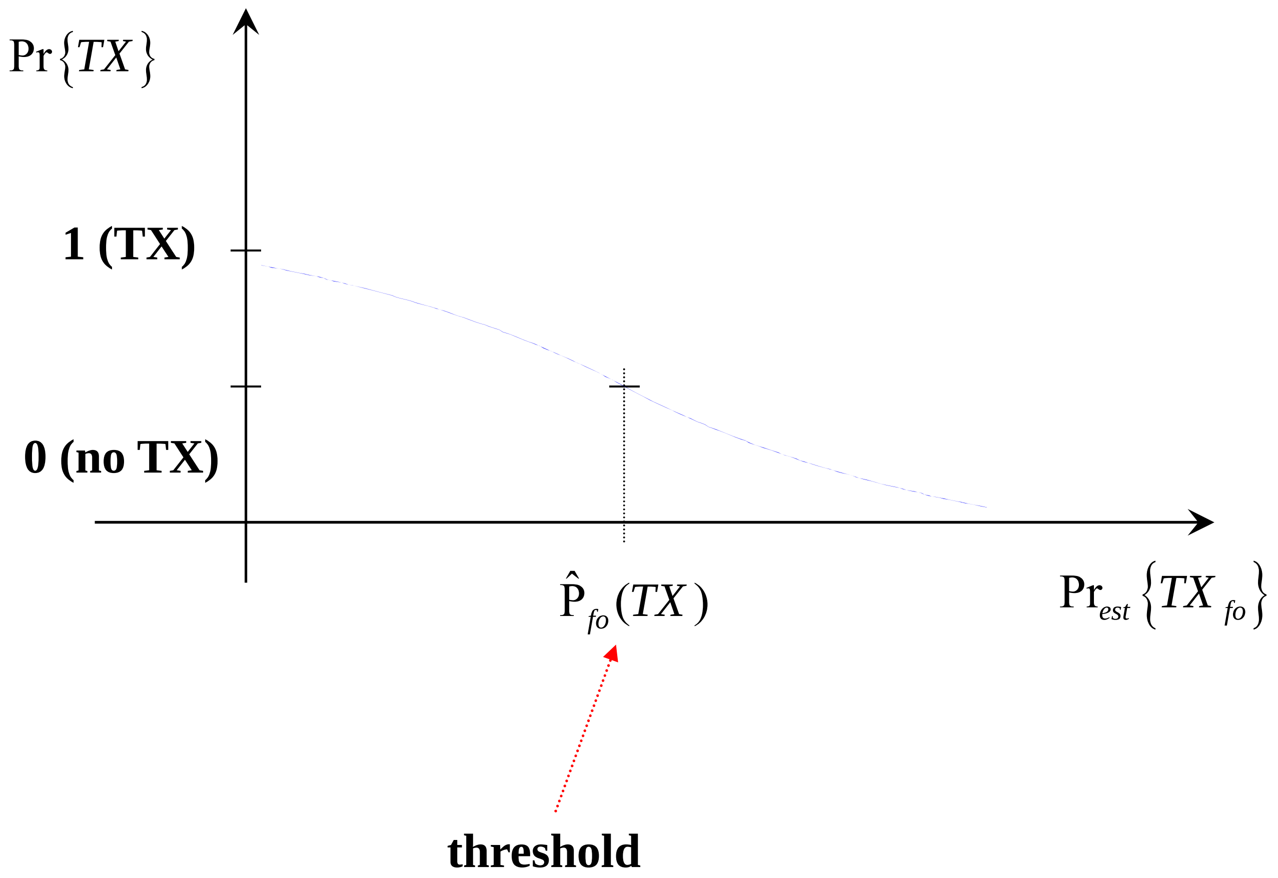
• In order to avoid a **discontinuous behavior** of the players, the adoption of a stochastic version of the considered fictitious game is recommended.

• This explains why the *smoothed best response* (SR) curve is introduced in our transmission strategy. Such a curve is defined by

$$\Pr\{TX\} = \begin{cases} 1 - \dfrac{\exp\left(-\gamma \Pr_{est}\{TX_{fo}\}\right)}{2\exp\left(\gamma \hat{\mathrm{P}}_{fo}(TX)\right)} & \Pr_{est}\{TX_{fo}\} \le \hat{\mathrm{P}}_{fo}(TX) \\ \dfrac{\exp\left(-\gamma \Pr_{est}\{TX_{fo}\}\right)}{2\exp\left(-\gamma \hat{\mathrm{P}}_{fo}(TX)\right)} & \Pr_{est}\{TX_{fo}\} > \hat{\mathrm{P}}_{fo}(TX) \end{cases}$$

and consists of two exponential pieces connecting at the *indifference point* $\hat{\mathrm{P}}_{fo}(TX)$.

Solving the game



Solving the game

- Note that:

1) the factors

$$\left[2\exp\left(\gamma\,\hat{\mathrm{P}}_{fo}(TX)\right)\right]^{-1} \quad \text{and} \quad \left[2\exp\left(-\gamma\,\hat{\mathrm{P}}_{fo}(TX)\right)\right]^{-1}$$

normalize the smoothed curve;

2) the parameter $\gamma \in \Re^{+}$ provides a degree of freedom for a proper adjustment of the approximation of the smoothed best response curve to the discontinuous BR.

Convergence of the solution

• In a smooth fictitious game characterized by two pure equilibria and by one mixed equilibrium, the player strategy converges to one of the strategies associated with the pure equilibria with unitary probability (the final strategy depends, however, on the initial conditions of the game).

• The mixed strategy deleloped above is not associated with a stable equilibrium but, despite this, it can be deemed an acceptable solution, since the game evolves over time. In fact, the scenario considered in this work is time-varying, in the sense that the channel gains experienced by network nodes (and, hence, their payoffs) change over time.

Convergence of the solution

• Then, even if the behavior of each player evolves towards one of the strategies associated with a pure equilibrium, this attracting point is continuously changing.

• In this process the more profitable solution of the network is always followed even if, to allow a proper game update when the environmental conditions change, the adaptation proneness is reduced by a sufficiently large degree of smoothing.

Numerical results

- The performance of the proposed transmission strategy has been assessed for a double hop relay network containing **10 potential relay nodes**.

- **Assumptions**:

a) the time slot duration T_U is known (and common) to all the network nodes;

b) the wireless link between any couple of nodes is affected by time-selective Rayleigh fading with Doppler bandwidth B_D (the well known Jakes' model has been used in our simulations) and the channels affecting distinct links are statistically independent;

c) the values $N_{moves} = 10$ and $\gamma = 10$ have been selected empirically;

d) the linear models $f(x) = x$ and $g(x) = k\,x$ have been adopted for the functions introduced previously. The value selected for the parameter k ensures that the two terms range over similar intervals and, consequently, influence the payoffs in a comparable fashion.

Numerical results

- Our transmission strategy has been compared with the opportunistic transmission selection scheme of [1] and with a simple symmetric contention channel access protocol [2].

[1]A. Bletsas, A. Khisti, D. P. Reed and A. Lippman, "A simple Cooperative diversity method based on network path selection", *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 659 - 672, Mar. 2006.

[2] A.S. Tanenbaum, **Computer Networks**, Printice Hall, 2003.

Numerical results

• In the first scheme each potential relay initializes a timer with a value which is inversely proportional to the estimated channel gain any time a data unit is ready to be forwarded; consequently, the timer of the most suitable relay decreases to zero more quickly.

• In the adopted symmetric contention protocol any node is supposed to know the number N of potential relays and transmits with a fixed probability (equal to $1/N$) in each time slot.

Computer simulations have been run to assess:

1) the *average throughput*

$$th \triangleq \frac{N(TX)}{T}$$

of the considered transmission strategies, where $N(TX)$ is the number of packets correctly transmitted by the whole relay stage and T is the number of time units considered in the simulations;

2) the *energy efficiency*

$$eff \triangleq \underset{n\in CL}{E}\left(\frac{N\left(TX_{all}(n)\right)}{E_n}\right)$$

of the transmission schemes. Here *n* is the index selecting the node in the cluster CL of potential relays, and $N\left(TX_{all}(n)\right)$ and E_n are the number of transmission attempts of the *n*-th node and the overall energy spent by the same node, respectively. For a given *n* the parameter E_n is evaluated summing up the quantities

$$E_n = \frac{1}{\left|h_{n,\hat{t}}\right|^2}$$

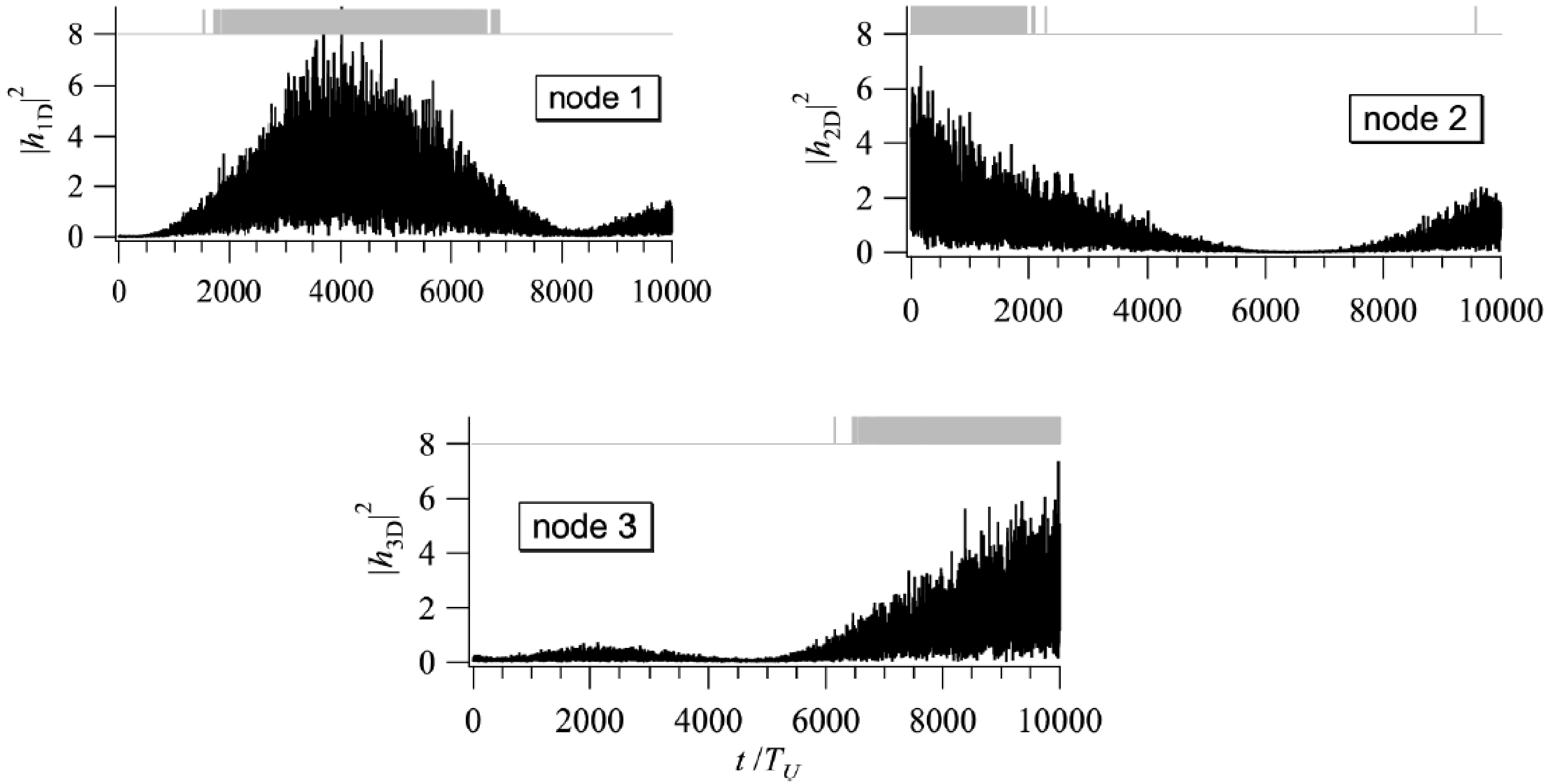
$\hat{t}$ = 1, 2, ..., where $h_{n,\hat{t}}$ is the complex channel gain experienced by the node *n* over the $\hat{t}$-th time slot.

Numerical results

• It is worth noting that the energy efficiency allows to assess the ability of the transmission strategy in exploiting the best options within the pool of available channels at the potential relays.

• The following figure shows some randomic channel realizations and the actions selected by each node in a simple experiment characterized by 3 potential relays. In this representation each node decides to transmit (to remain silent) when its boolean indicator is equal to 1 (0). This figure evidences the rationality of the proposed transmission strategy, since it shows that a) an order can emerge in the packet transmissions of the nodes even if there is not any explicit negotiation among them and b) the potential relay offering the best communication channel is always able to exploit it.

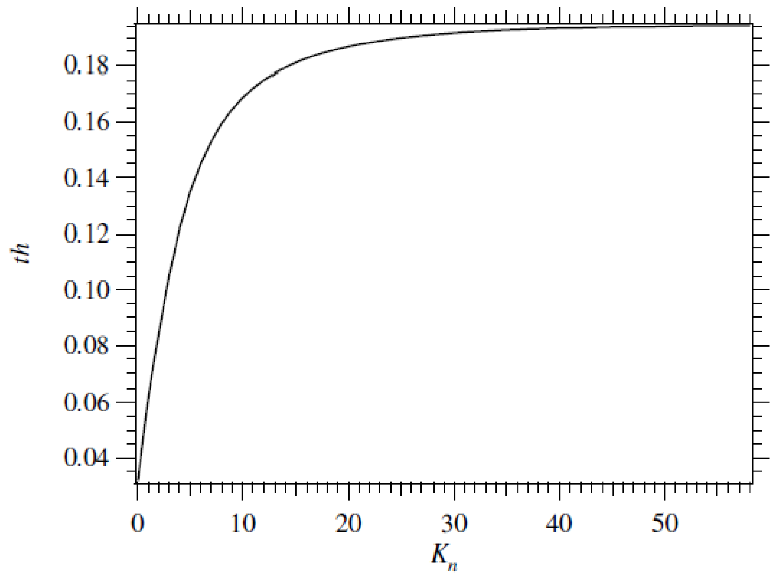
Numerical results



Channel power gains experienced by 3 distinct nodes in a cluster and their transmission attempts.

Numerical results

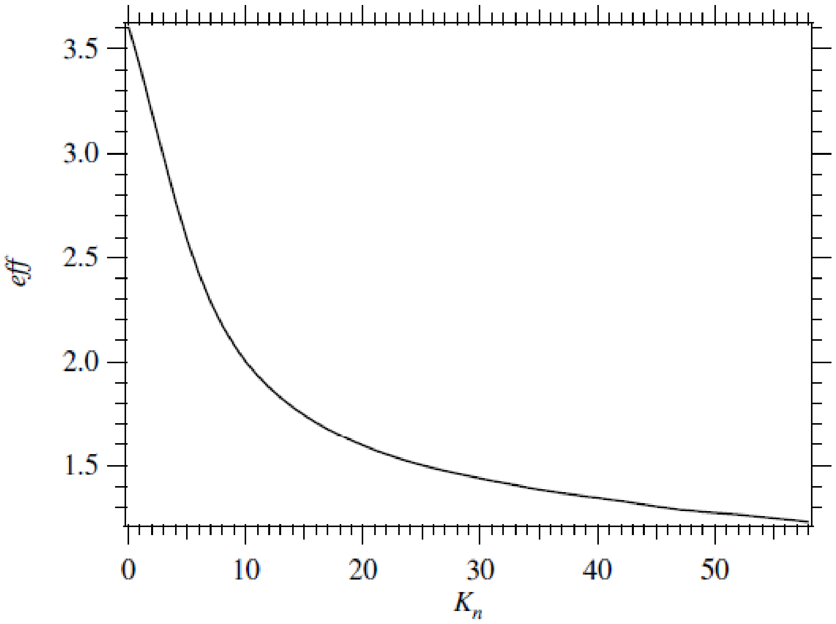
• The following figure illustrates the mean throughput achievable by a link exploiting the game-based transmission strategy versus the risk affinity factor K_n . These results show that an increase of K_n leads to a larger throughput, even if the growth rate becomes gradually smaller and then stabilizes because of a rise in the number of collisions.



Achievable throughput versus the mean risk affinity of the cluster nodes for the proposed transmission strategy.

Numerical results

• It is also worth pointing out that, as evidenced by the following figure, an excessive increase of K_n is damaging for the energy efficiency of the link because it leads to frequent collisions

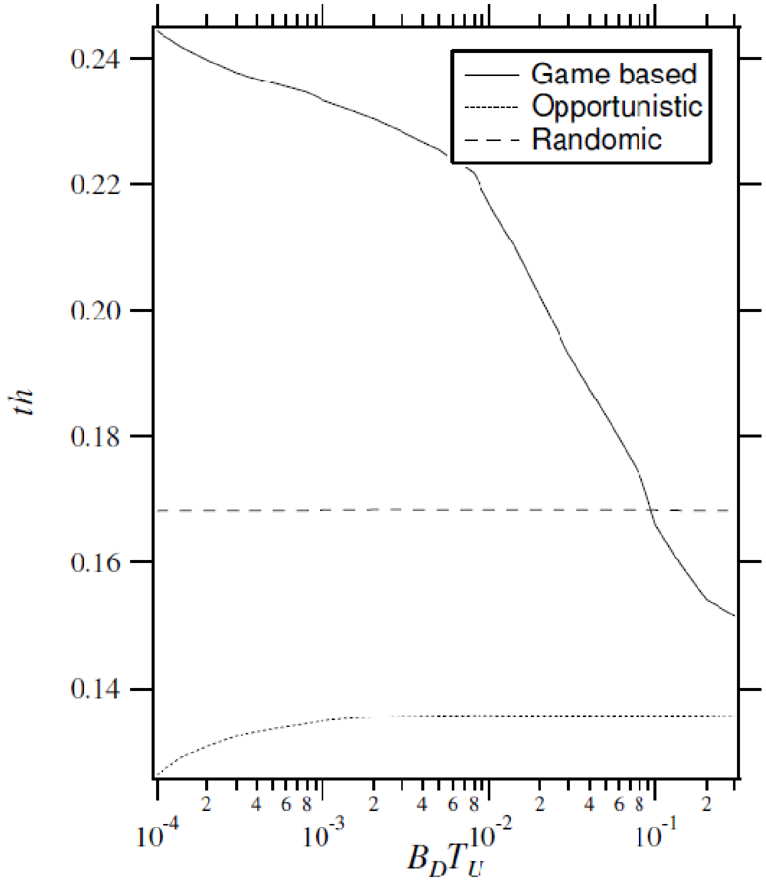


Energy efficiency of the link versus the mean risk affinity of the cluster nodes for the proposed transmission strategy.

Numerical results

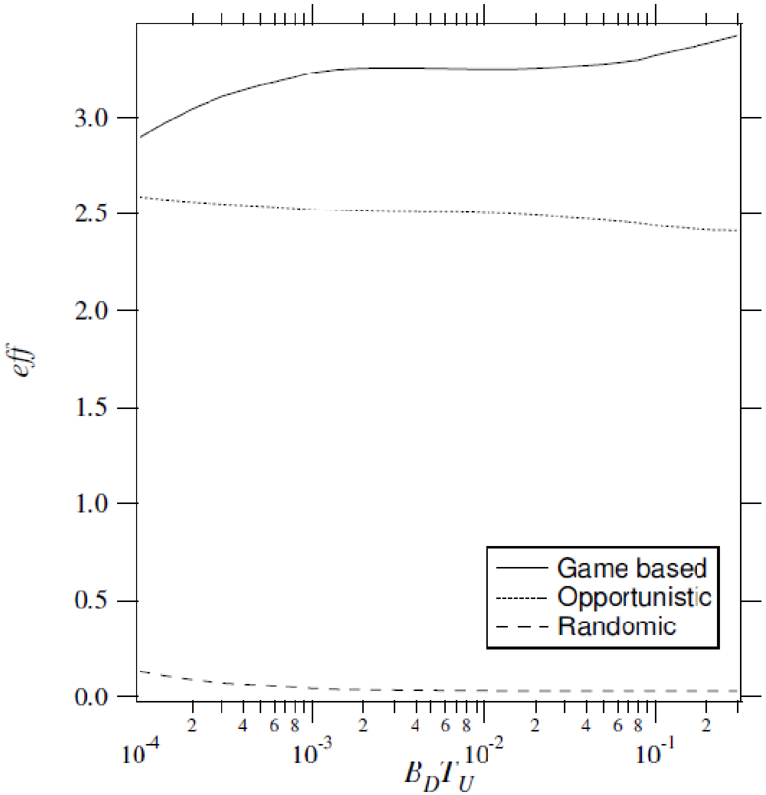
• The following two figures compare the above mentioned three transmission schemes in terms of mean achievable throughput and energy efficiency versus the normalized Doppler bandwidth $B_D T_U$ and for a fixed K_n . These results evidence that the proposed approach substantially outperforms the opportunistic strategy without increasing the number of collisions (their presence would reduce the energy efficiency of the link). This performance gap is due to the fact that the opportunistic solution is penalized by the variable delay characterizing the transmission of the node with the best channel.

Numerical results



• Comparison among the achievable throughputs (versus the normalized Doppler bandwidth $B_D T_U$) offered by three different transmission strategies.

Numerical results



• Comparison among the energy efficiencies (versus the normalized Doppler bandwidth $B_D T_U$) offered by three different transmission strategies.

Numerical results

• These results also show that the throughput offered by the symmetric contention protocol is lower than that of the approach we propose and is characterized by a larger number of collisions, resulting in a decrease of the energy efficiency. The throughput offered by the proposed solution approaches one half of the maximum achievable throughput on the link (which is is equal to 0.5 for a double hop link based on half duplex nodes) and decreases significantly only in the presence of very fast fading (say, when $B_D T_U$ approaches 0.1), since the correlation between subsequent game turns reduces and so also the effectiveness of the learning strategy to face the moves selected by the opponent.

Conclusions

• Game theory has been applied to develop a novel cooperative transmission strategy for data communications in an ad-hoc wireless network.

• This strategy is functionally equivalent to a transmission selection scheme, which is managed, however, in a fully distributed fashion. The proposed strategy consists of an autonomous choice, made by each potential relay in a cluster of nodes, between two simple alternatives: transmitting an information data packet to a destination or remaining silent.

• This allows to coordinate the transmissions among the potential relays without any explicit information exchange between them. Thanks to this feature, the proposed solution offers a larger throughput and higher efficiency than other communication techniques exploiting distributed transmission selection.

A Game Theoretical Approach to the Management of Node Clusters in DR-OSTC Based MISO Cooperative Communications

Introduction

• In wireless ad hoc and sensor networks data communications usually require the *cooperation* of their nodes; for instance, data transmission from a given source node to a far destination node can involve other nodes acting as **relays** to establish a reliable and energy efficient *multihop link.*

• To enhance link performance in each hop, relays can be also grouped to form **clusters** of cooperative transmitters; when this occurs, the nodes of each cluster coordinate their data transmissions according to a specific strategy, i.e. according to a given **distributed cooperative transmission technique**.

• An important example of this approach is offered by the so called *distributed space-time coding schemes*, like **distributed orthogonal space-time coding** (D-OSTC) [1].

[1] **J. N. Laneman and G. W. Wornell**, "Distributed Space-Time-Coded Protocols for Exploiting Cooperative Diversity Wireless Networks", *IEEE Trans. Inf. Theory*, vol.49, no.10, pp.2415-2425, Oct. 2003.

Introduction

• Unluckily, the implementation of distributed transmission methods is hindered mainly by their significant complexity and by the large overhead required for node management.

• This is due to the need of identifying the nodes that can potentially join each cluster and of assigning the available codewords to cluster nodes in a proper fashion.

• Recently, a solution to the problem of codeword assignment for D-OSTC has been proposed in [2], [3]. According to this solution, known as *distributed randomized – orthogonal space-time coding* (DR-OSTC), each node transmits a linear combination of multiple codewords, which are randomly selected from a code matrix shared by all the network nodes.

[2] B. Sirkeci-Mergen and A. Scaglione, “Randomized distributed space-time coding for cooperative communication in self organized networks”, *Proc. IEEE Workshop on Signal Processing Advances in Wireless Communications 2005* (SPAWC 2005), pp. 500-504, June 2005.
[3] B. Sirkeci-Mergen and A. Scaglione, “Randomized Space-Time Coding for Distributed Cooperative Communication”, *IEEE Trans. Signal Proc.*, vol.55, no.10, pp. 5003-5017, Oct. 2007.

Introduction

• In principle, the DR-OSTC scheme does not require an a priori knowledge of the number of nodes contributing to data transmission in each cluster.

• In practice, however, an accurate code design and a proper number of nodes in each distributed transmission are required to ensure a given **outage probability**.

• These problems are analysed in detail in [4], where the minimum number of nodes required to accomplish a cooperative transmission and the code size needed to satisfy given performance requirements are derived.

• However, as fas as we know, what is still missing in the technical literature about DR-OSTC is **an efficient and low overhead strategy for the selection of a proper set of nodes for cooperative transmissions**.

[4] S. Savazzi, U. Spagnolini, "Distributed Orthogonal Space-Time Coding: Design and Outage Analysis for Randomized Cooperation", *IEEE Trans.Wireless Commun.*, vol. 6, no. 12, pp. 4546 - 4557, Dec. 07.

Introduction

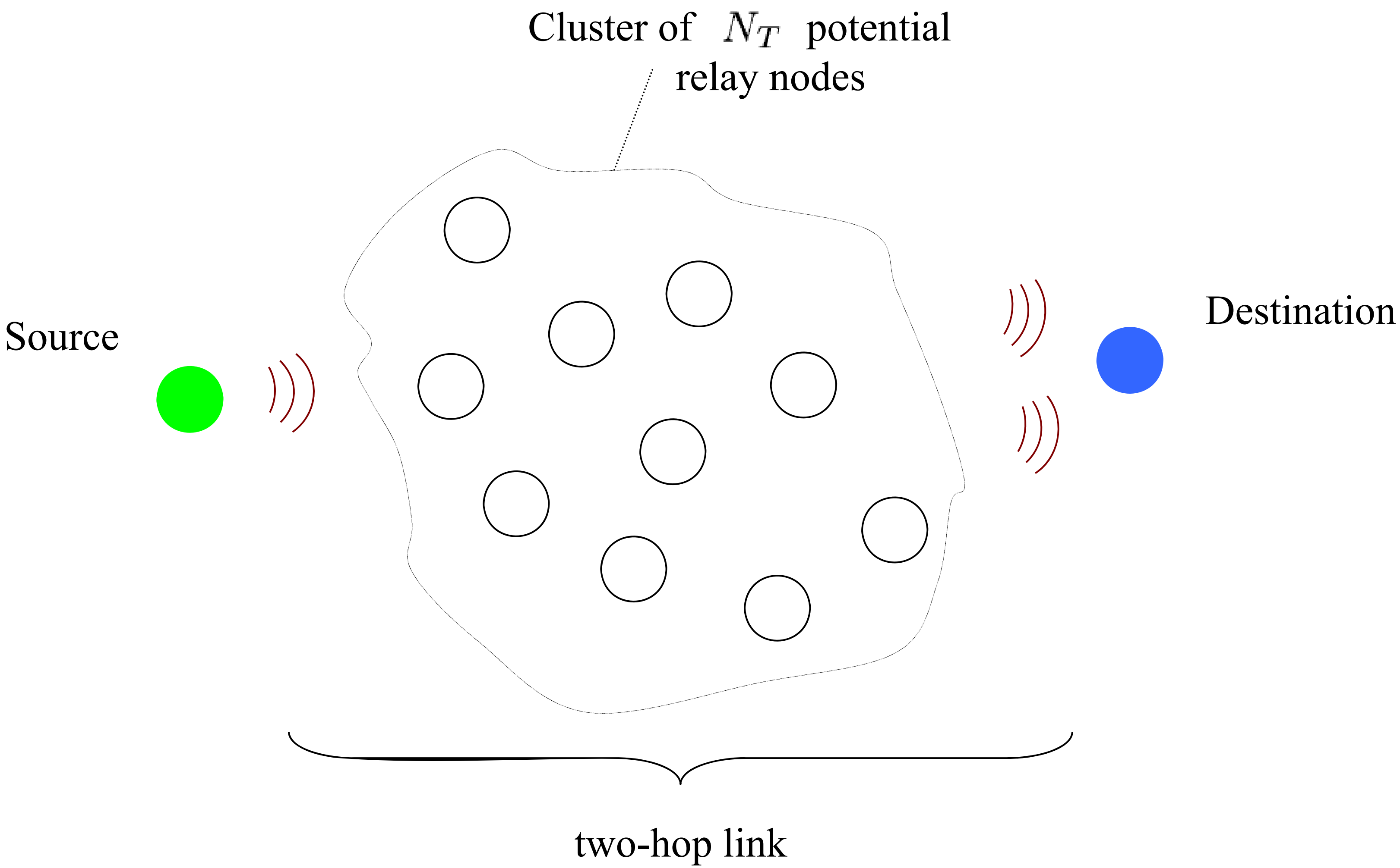
• In the following a **novel distributed approach to the management of node participation** in a DR-OSTC based cooperative link is derived resorting to a **game theoretical modelling** of the considered problem [5].

• In a finite *signal-to-noise ratio* (SNR) scenario, the proposed strategy is able to reach the needed diversity order on a cooperative link and, at the same time, to avoid an energy waste deriving from an excessive number of cooperative nodes.

• In addition, it is characterized by the following relevant features:
a) each node is allowed to manage, in a completely autonomous fashion, its contribution within a cluster of potential relays;
b) no transmission overhead is required for node management;
c) no prior information about the distribution of neighbouring nodes or the channel statistics are needed for the distributed management of network nodes.

[5] S. Sergi and G. M. Vitetta, "A Game Theoretical Approach to Distributed Relay Selection in Randomized Cooperation", *submitted to the IEEE Transactions on Wireless Communications*.

Network model



Network model

• The potential relay nodes are hierarchically equivalent, rational, are endowed with a single antenna and operate in a *decode and forward* fashion.

• Each node is also equipped with a battery having a finite stored energy.

• Each data packet sent by the source can be correctly detected by a number of potential relays (correct detection depends on both transmission power and channel state); then each relay can decide if forwarding the packet towards the destination or not. For energy saving it is advisable to let a proper number of relay nodes contribute to packet forwarding.

• As shown in the following, this goal can be achieved adjusting the *transmission probability* P_{tx} of the potential relay nodes, so that a subset of nodes (selected in the set of N_T available nodes) actually plays, on the average, an active role in the task of cooperative forwarding.

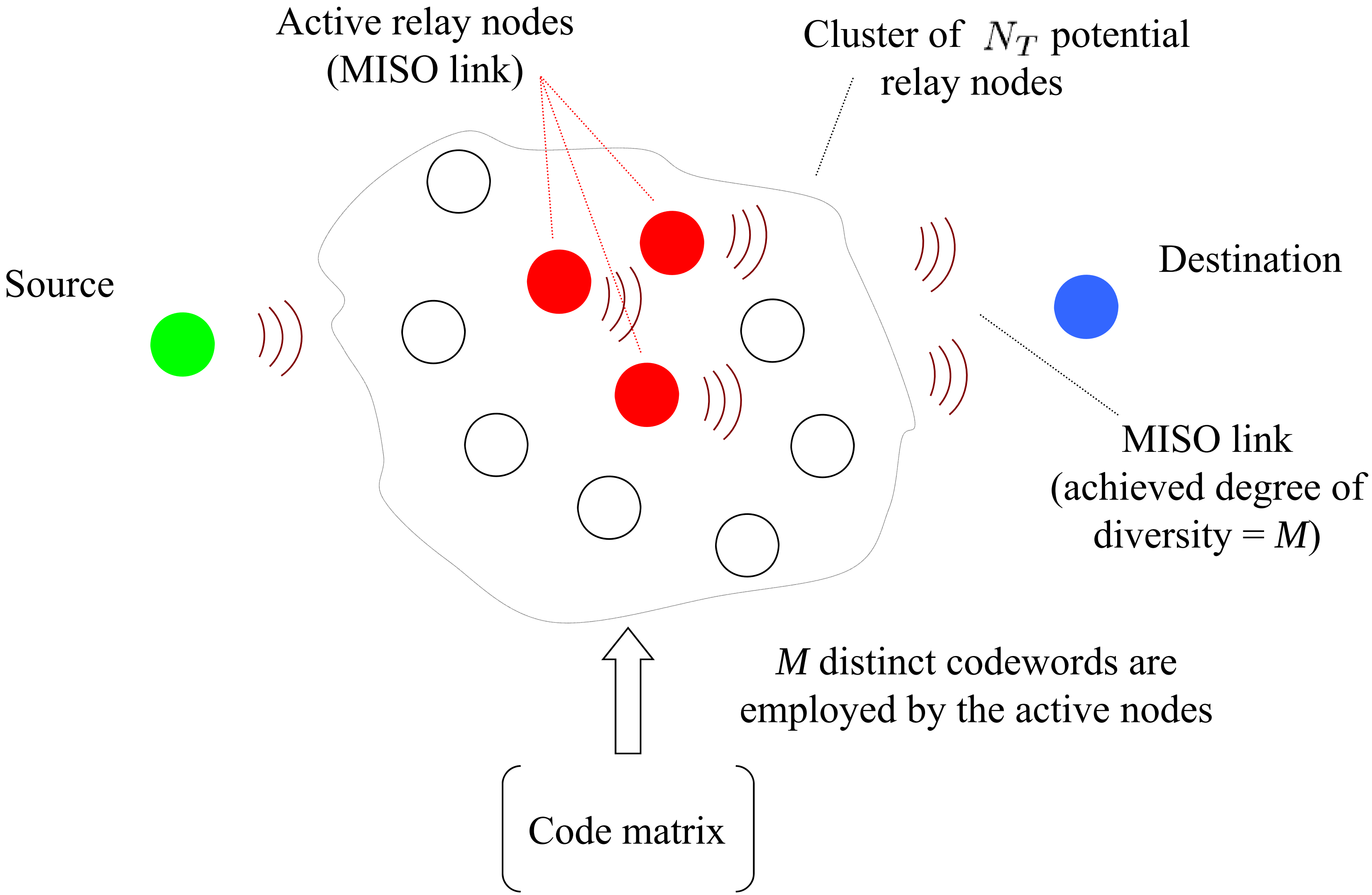
Network model

• The active nodes adopt the DR-OSTC scheme proposed in [2] for data transmission; this means that each node uses a single codeword randomly selected from a common code matrix of proper size for data relaying.

• It is well known that, in this scenario and in finite SNR conditions, the **achievable performance on a link mainly depends on the number *M* of distinct codewords employed by at least one node**.

• For this reason, in the following the performance enhancement deriving from the exploitation of the same codeword from multiple transmitting nodes of the same cluster is neglected. Moreover, to simplify our analysis, the errors due to channel impairments (hence fading and noise) are also neglected in the relays to destination *multiple input – single output* (MISO) link, so that the only figure of merit taken into account when assessing link quality is represented by the **achieved degree of diversity**.

Network model



Network model

• **Further assumptions**:

a) The packets transmitted by the source contain a **known preamble** which can be exploited by all the potential relays to achieve a rough synchronization for their transmission.

b) The whole set of potential relays is always of **adequate size**, so that a reliable data transmission can be accomplished.

c) The nodes belonging to the same potential relay cluster **do not exchange information**, but are able to listen to a common signal, originating from the destination node, and carrying information about the status of the last transmission attempt (single bit ACK/NAK feedback).

d) The **time axis is divided in slots** to ease the modellization of node actions. The slot length T_u is equal to the duration of a data packet transmitted by network nodes. Note that, generally speaking, the duration of the slot period can appreciably influence system performance in the presence of a time varying wireless channel.

Network model

- ***Scenario* #1**: all the nodes are ***selfish***. If we take into consideration the general case of an ad hoc network consisting of peer nodes, the first scenario refers to the situation in which each user owns its terminal and aims only at carrying out its own data communications; in this case, in the eyes of every potential relay, any cooperation effort is perceived as a waste of personal resources so that, generally speaking, it has to be properly stimulated.

- ***Scenario* #2**: all the nodes are ***prone to cooperation***. This scenario is well suited to describe, for example, sensor networks, where the nodes are under the control of the same central authority. In fact, in the last situation the only goals of each node are the efficiency and the effectiveness of the network it belongs.

Game description

• **Our main goal is achieving a degree of diversity *M* equal to *R* at the destination node**, so that data decoding can be carried out correctly without involving an excessive number of active relays (and a consequent energy waste).

• To reach our goal, we need to devise a **node management strategy** such that, if a packet is received by a potential relay cluster consisting of N_T nodes, an adequate number $N(t)$ of them decides to contribute to data relaying in the *t* - th slot.

• Moreover, we are interested in devising a ***distributed and noncooperative strategy***, so that any explicit information sharing among the N_T nodes is avoided.

Game description

- To achieve this target we model the *participation dilemma* (i.e., joining or not the set of nodes accomplishing a cooperative transmission) as a ***multiplayer game***.

- In such a game the **set of players** consists of the nodes belonging to the potential relay cluster and the **action set** of each player is made of two distinct options, namely transmitting or remaining silent. The **payoffs** associated with the node actions depend on the node behaviour.

- **Scenario #1 (selfish nodes) -** The benefit acquired by the *n*-th node is related to its **active contribution within a successful cluster transmission** and is **inversely proportional to the number of cooperating relays**; in addition, a **cost related to the energy spent for packet transmission** (this depends on the currently experienced channel condition) is charged to each node irrespectively of the transmission success.

Game description

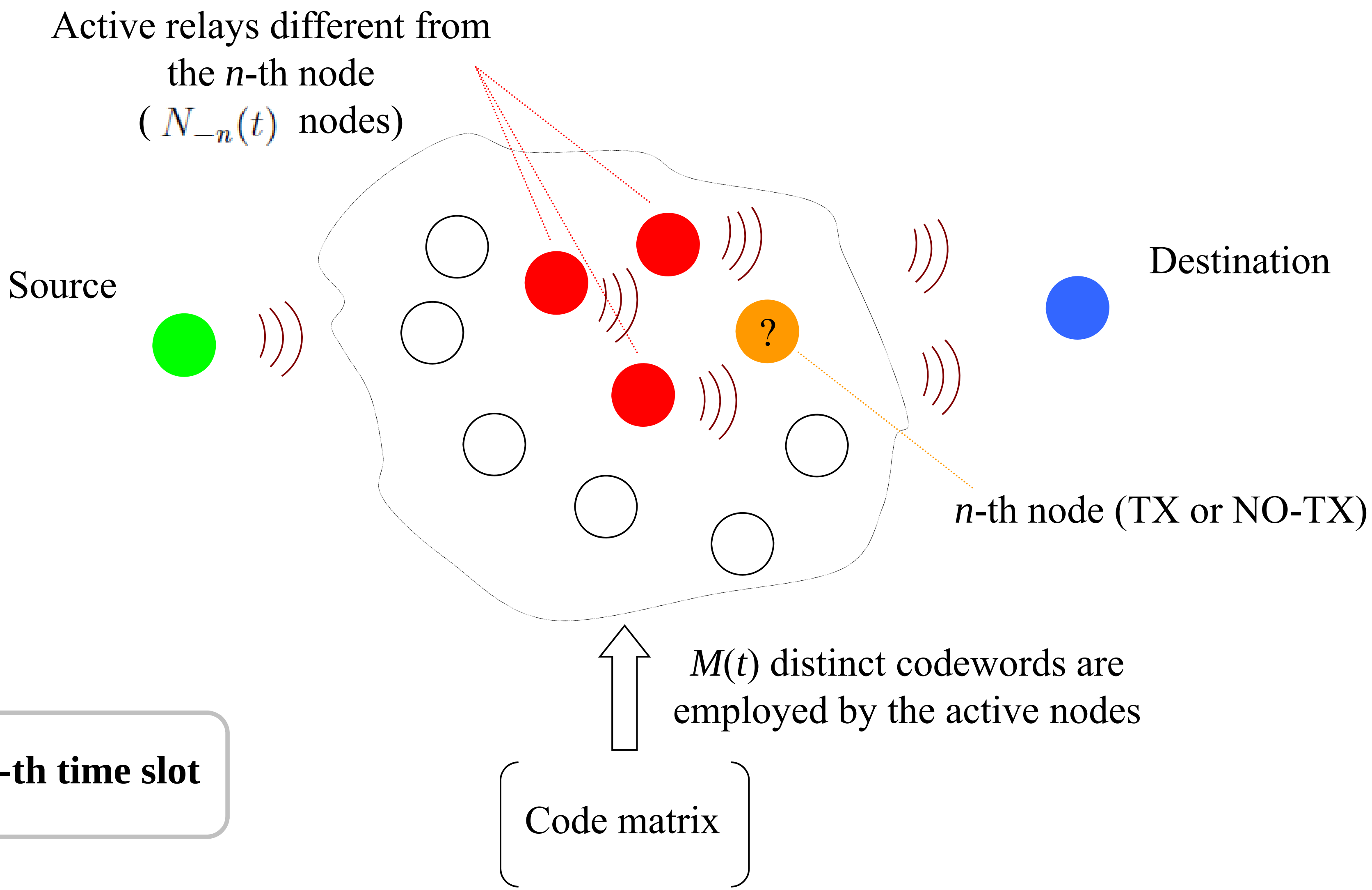
- For these reasons the following rules are established for the **payoffs**:

a) If the *n*-th node **decides to take part** to a cooperative transmission of a data packet towards the destination (TX action) and the **cluster it belongs to carries out this task correctly**, the associated payoff for this node is equal to a_n .

b) If the *n*-th node **decides to take part** to a cooperative transmission of a data packet towards the destination (TX action), but the **cluster it belongs to does not carry out this task correctly**, its payoff is equal to c_n (this denotes a waste of resources).

c) If the *n*-th node decides **not to join** the cooperative transmission of a data packet towards the destination (NO TX action) its payoff is equal to 0 **regardless of the actual transmission outcome**.

Game description



***t*-th time slot**

Game description

• The derivation of the **optimal transmission strategy** for the *n*-th node requires analysing the node point of view on its participation dilemma and, in particular, its *subjective* vision of the multiplayer game described above.

• This game can be usefully represented in *normal form*, so that the earnable payoffs can be easily identified on the basis of the actions selected by the *n*-th node and its opponents: such a representation is provided in the following Table.

Number of other transmitting nodes / n-th node action	$N_{-n}(t) \leq R-2$	$N_{-n}(t) = R-1$	$N_{-n}(t) \geq R$
TX	c_n	$\Pr\{M = R\}a_n + \Pr\{M < R\}c_n$	$\Pr\{M \geq R\}a_n + \Pr\{M < R\}c_n$
NO_TX	0	0	0

TABLE I

REPRESENTATION OF THE PARTICIPATION GAME FOR THE n-TH NODE IN SCENARIO #1

Game description

• Unluckily, the game we are analysing is characterized by a set of ***incomplete information*** about $N_{-n}(t)$; this prevents us from computing the expected payoff for the two different choices the *n*-th node can make. In particular, it is worth nothing that in scenario #1 the **payoff expected by the *n*-th node for a packet transmission** is given by

$$\rho(N_{-n}(t)) = \Pr\left\{M(t) \geq R | N_{-n}(t)\right\} a_n + \Pr\left\{M(t) < R | N_{-n}(t)\right\} c_n$$

- The *n*-th node needs to optimize its response to the probability mass function $\Pr\left\{N_{-n}(t)\right\}$, so that its mean payoff is maximised.

- For this reason, we consider the payoff metric

$$\alpha(N_{-n}(t)) = \Pr\left\{N_{-n}(t)\right\} \rho(N_{-n}(t)),$$

which represents a **weighted** version of the expected payoff defined above.

Game description

• The **goal of the *n*-th** node is to behave according to a strategy (hence, in our case, to choose its transmission probability $P_{tx}(n,t)$ in the *t*-th time slot) such that its **mean expected payoff**

$$E(TX) = \sum_{N_{-n}} \alpha(N_{-n}(t))$$

is maximized.

• The dependency on $N_{-n}(t)$ makes the derivation of an equilibrium point for the game a non trivial task. To tackle this problem, it is possible to consider the participation dilemma as a ***repeated game***. In fact, the game is continuously repeated during the life of a given wireless link since the *n*-th node has to take a decision about cooperating or remaining silent in each time slot. If all the payoffs can be deemed constant over a few turns of the game itself, the game can be deemed ***stationary*** and this allows the *n*-th node to acquire information about the behavior of its opponents from their past moves.

Game description

- Note also that the payoffs shown in Table I depend on the probability of the events $\{M(t) < R\}$ and $\{M(t) \geq R\}$, i.e. on the probability that the available degree of diversity is smaller than that needed for a correct transmission or not, respectively.

- It is easy to understand that these probabilities depend both on the overall number $N(t)$ of nodes involved in a cooperative transmission and on the randomization rule adopted for the codeword assignment. If L distinct codewords are available and a *randomic codeword selection* is assumed, closed form expressions to evaluate these probabilities can be derived.

$$N(t) = \begin{cases} N_{-n}(t) & \text{if NO-TX from the } n\text{-th node} \\ N_{-n}(t)+1 & \text{if TX from the } n\text{-th node} \end{cases}$$

Evaluation of the payoffs

Payoff evaluation in scenario #1

• In the following we assume that the **overall amount of credits** earned by the *n*-th node thanks to its transmissions until the end of the *t*-th slot over a specific link is equal to

$$P_{ov}(n,t) = B\frac{F_w(n,t)}{\sum_{j\in C_T} F_w(j,t)},$$

where *B* is the overall amount of credits made available by the source node to reward the whole potential relay set, $F_w(n,t)$ is the number of packets that the node has contributed to forward until the end of the *t*-th slot and $\sum_{j\in C_T} F_w(j,t)$ is the overall number of packets sent by the whole potential relay set C_T, which the *n*-th node belongs to, over the same time interval.

Evaluation of payoffs

• This choice ensures that the source node can define a priori the amount of credits *B* to be assigned to a relay stage to gain its cooperation, independently of the fraction of nodes that will really contribute to a packet transmission on the second hop. Moreover, it will contribute to limit the overall number of potential relays cooperating in a successful transmission, so that the efficiency of the link itself is ensured.

• For all the assumptions made above, if $E_{ov}(n,t)$ denotes an estimate of the overall energy consumed by the *n*-th node until the end of the *t*-th slot, **the overall benefit acquired by the node over the given time interval** can be expressed as

$$f\left(P_{ov}(n,t)\right) - g\left(E_{ov}(n,t)\right),$$

where $f(\cdot)$ and $g(\cdot)$ are monotonic increasing functions having the specific purpose of making the two terms of the last equation *homogeneous* and characterized by similar ranges (so that they play comparable roles in the evaluation of payoffs).

Evaluation of payoffs

- Since the overall benefit for the *n*-th node after **a correct transmission in the (*t* + 1)-th slot** is given by

$$f\left(P_{ov}(n, t+1)\right) - g\left(E_{ov}(n, t+1)\right),$$

the payoff a_n (referring to the (*t* + 1)-th slot) can be defined as

$$a_n \triangleq \left[f\left(P_{ov}(n, t+1)\right) - g\left(E_{ov}(n, t+1)\right)\right]$$
$$\left[f\left(P_{ov}(n, t)\right) \quad g\left(E_{ov}(n, t)\right)\right],$$

since this expresses the additional benefit acquired by the *n*-th relay node for the transmission of the new packet. This definition deserves the following comments: a) it implies that $a_n > 0$ ($a_n < 0$) corresponds to an **actual reward** for the node (a **damage** for it); b) it does not take into account the **willingness** of the *n*-th node to spend its residual resources for data transmission (hence, to earn credits for its future needs).

Evaluation of payoffs

- To circumvent the last problem, the last expression can be generalized as

$$\hat{a}_n \triangleq \left[w_{tx}(n) \cdot f\left(P_{ov}(n,t+1)\right) - w_{en}(n) \cdot g\left(E_{ov}(n,t+1)\right)\right]$$

$$- \left[w_{tx}(n) \cdot f\left(P_{ov}(n,t)\right) - w_{en}(n) \cdot g\left(E_{ov}(n,t)\right)\right],$$

where the weight $w_{tx}(n)$ ($w_{en}(n)$) measures the willingness of the *n*-th node to cooperate (to save energy). Note that, since the functions *f*($\cdot$) and *g*($\cdot$) are generic, the real influence of these weights on the payoffs depends on **their ratio**, i.e. on the parameter

$$K_n \triangleq \frac{w_{tx}(n)}{w_{en}(n)},$$

which can be interpreted as a *risk affinity* for the *n*-th node. A large risk affinity pushes the *n*-th node to cooperate with the aim of gaining credits to acquire the neighbour's support in the near future; a small risk affinity, instead, can be interpreted as an appreciable energy avidity, which pushes the node to cooperate scarcely and only when its channel conditions are favourable.

Evaluation of payoffs

- The payoff c_n assigned to the *n*-th node for an **unsuccessful transmission** in the (t + 1)-th slot can be evaluated resorting to the approach described for a_n . We have that

$$c_n \triangleq g\left(E_{ov}(n,t)\right) - g\left(E_{ov}(n,t+1)\right)$$

$$\hat{c}_n \triangleq w_{en}\left(n\right) \cdot \left[g\left(E_{ov}(n,t)\right) - g\left(E_{ov}(n,t+1)\right)\right]$$

- Note that the payoff $\hat{c}_n$ is insensitive to the risk affinity factor K_n .

Strategy played by the opponents

• The derivation of the optimal transmission strategy to be played by the *n*-th node of a relay cluster in the *t*-th slot requires an estimate of the probability $\Pr\{N_{-n}(t)\}$.

• This knowledge can be acquired through a **proper learning strategy** that takes advantage of the repetitiveness of the game itself; in our derivation it is assumed that the degree of diversity *R* needed on a wireless link is perfectly known to the potential relay nodes.

• It can be shown that probability $\Pr\{N(t)\}$ can be estimated under the following assumptions:

1) $M(t)$ is described by a **Poisson model**; the unknown parameter $\lambda_M(t)$ of this model can be estimated from the observation of the last T_0 transmission attempts made by the relay cluster.

Strategy played by the opponents

2) The regime in a normal functioning of our system is characterized by a **Gaussian shaped probability mass function** for $N(t)$ (the mean value $\bar{N}(R)$ depends on L and R only)

- However, the random parameter of real interest in our game is the number $N_{-n}(t)$ of opponents only. If we refer to the last T_0 turns of the game, the *n*-th node contribution can be deemed **decisive** for the cluster it belongs to if the overall number of transmissions n_{TX} accomplished by the node itself is not smaller than $T_0/2$; otherwise it is **marginal**. For this reason the estimate

$$\Pr\{N_{-n}(t)=n\}=\begin{cases}\Pr\{N(t)=n+1\} & if\ n_{TX}\geq T_0/2\\ \Pr\{N(t)=n\} & otherwise\end{cases}$$

is adopted. Thanks to the hypothesis of stationarity, this result can be deemed stable in the time interval of a few consecutive game turns; therefore, it can be exploited to **estimate the number of opponents that will transmit in the next repetition of the game**.

Node participation strategy

• Generally speaking, whatever the scenario, the goal of the *n*-th node is to adjust its transmission probability $P_{tx}(n,t)$ in the *t*-th slot in order to maximise its **mean expected payoff**

$$E(TX) = \sum_{N_{-n}} \alpha(N_{-n}(t))$$

• Therefore the goal of the *n*-th node becomes the maximization of the ratio between the positive and the negative areas of the weighted expected payoff curve. To achieve this target, starting from an **initial strategy** that establishes a transmission probability equal to $P_{tx}(n, t=0)$, the node is induced to **adapt its transmission probability**, in order to contribute to modify the probability $\Pr\{N(t)\}$, which currently describes the **global strategy played by the relay stage**.

• This approach relies on the assumption that the weighted expected payoff curve does not substantially differ from node to node, so that all the potential relays are expected to favour a modification of their strategies with the common aim of increasing or decreasing the population of active transmitters.

Node participation strategy

- For this reason, it is advisable for the *n*-th node to contribute to a change of $\Pr\{N(t)\}$ proportionally to the gain it expects from this variation; this suggests to update its transmission probability as

$$P_{tx}(n,t) = P_{tx}(n,t-1) + \beta\left(\sum_{N_{-n}\in C_1} |\alpha(N_{-n}(t))| - \sum_{N_{-n}\in C_2} |\alpha(N_{-n}(t))|\right)$$

$C_1 \quad \Longrightarrow \quad \{N_{-n}(t) < \bar{N}(R) | \alpha(N_{-n}(t)) < 0\}$

$C_2 \quad \Longrightarrow \quad \{N_{-n}(t) > \bar{N}(R) | \alpha(N_{-n}(t)) < 0\}$

β step size

Node participation strategy

• The rationale behind the update equation is that the payoff earnable by the *n*-th node is positive within a certain (and dependent on its subjective channel experience) range of $N(t)$ around $\bar{N}(R)$; the values of $N(t)$ in this range allow the cluster to reach with high probability the needed degree of diversity and, at the same time, the *n*-th node to earn a fraction of the credits at disposal sufficient to cover the consumed energy.

• Therefore, given an imperfect estimate of $N(t)$, the goal of each node is to maximize its mean achievable payoff or, from a different perspective, to minimize the mean negative payoff that could arise from its transmission. In fact, the transmission probability is adjusted in a way to minimize the probability of the occurrence of the cases producing a negative value of the payoff $\alpha(N_{-n}(t))$ for the given node.

Node participation strategy

• The proposed participation strategy for the *n*-th node can be summarized in the following steps:

1. Computation of the expected payoffs (see Table I) resulting from a transmission attempt of the *n*-th node for all the possible values of $N_{-n}(t)$

2. **Estimation of the probability mass function** $\Pr\{N_{-n}(t)\}$ of the number of nodes transmitting in the following turn.

3. Computation of the metrics $\{\ \alpha(N_{-n}(t))\ \}$

4. Assessment of the node participation strategy based on

$$P_{tx}(n,t) = P_{tx}(n,t-1) + \beta\left(\sum_{N_{-n}\in C_1} |\alpha(N_{-n}(t))| - \sum_{N_{-n}\in C_2} |\alpha(N_{-n}(t))|\right)$$

Node participation strategy

• When considering **scenario #2**, the rationale described so far still holds. However, when the network is populated by nodes prone to cooperation, it is reasonable to assume that **each node earns a constant reward for a correct packet transmission of the cluster it belongs to, independently of its actual cooperation**.

• In other words, a node earns a positive payoff also when it decides not to join a cooperative transmission of a data packet towards the destination (NO TX action), but the active cooperating nodes correctly carry out this task.

Node participation strategy

Number of other transmitting nodes / n-th node action	$N_{-n} \leq R-2$	$N_{-n} = R-1$	$N_{-n} \geq R$
TX	$\acute{c}_n$	$\Pr\{M = R\}\acute{a}_n$ $+\Pr\{M < R\}\acute{c}_n$	$\Pr\{M \geq R\}\acute{a}_n$ $+\Pr\{M < R\}\acute{c}_n$
$NO\text{-}TX$	0	0	$\Pr\{M \geq R\}\acute{b}_n$

- Representation of the participation game for the *n*-th node in scenario #2

Node participation strategy

• In scenario #2 a different solution can be developed taking as an unknown parameter the ***degree of diversity*** achieved by the involved link (thanks to the nature of the employed payoffs).

	$M_{-n} \leq R-2$	$M_{-n} = R-1$	$M_{-n} \geq R$
TX	$\acute{c}_n$	$P_{(R-1)\to R}\ \acute{a}_n + \left(1 - P_{(R-1)\to R}\right) \acute{c}_n$	$\acute{a}_n$
$NO\text{-}TX$	0	0	$\acute{b}_n$

• Representation of the participation game based on the number of selected codewords for the *n*-th node in scenario #2

Node participation strategy

• Here, $M_{-n}(t)$ denotes the number of codewords selected by at least one transmitting node, other than the *n*-th one. All the payoffs appearing in the table of the previous slide are defined except that referring to the event of transmission with

$$\{M_{-n}(t) = R - 1\}$$

• This occurs with probability

$$P_{(R-1)\to R} = \frac{(L - (R-1))}{L}$$

• The *n*-th node transmits a data packet if the expected payoff coming from this action is larger than that it can earn if it remains silent. These payoffs are given by

Node participation strategy

$$E(TX) = \acute{c}_n \Pr\{M_{-n}(t) \leq R-2\} + \acute{a}_n \Pr\{M_{-n}(t) \geq R\} + \left(\begin{array}{c} \acute{a}_n P_{(R-1)\to R} + \\ \acute{c}_n (1 - P_{(R-1)\to R}) \end{array} \right) \Pr\{M_{-n}(t) = R-1\}$$

for the case of transmission and

$$E(NO\_TX) = \acute{b}_n \Pr\{M_{-n}(t) \geq R\}$$

in the opposite case. At the equilibrium point these two quantities have to be equal, hence

$$E(TX) - E(NO\_TX) = 0$$

Node participation strategy

• Therefore, the equilibrium point for the game that is characterized by the probability

$$\Pr\{M_{-n}(t) = R-1\} = -\frac{\acute{c}_n}{\acute{a}_n P_{(R-1)\rightarrow R}}$$

• The n-th node strategy is related to what it can infer from the current actions of its opponents.

• An estimate

$$\hat{\Pr}\{M_{-n}(t) = R-1\}$$

of the probability $\Pr\{M_{-n}(t) = R-1\}$ can be easily obtained from the observation of the last T_0 transmission attempts

Node participation strategy

• The best response for the *n*-th node is represented by cooperating if

$$\hat{\Pr}\left\{M_{-n}(t) = R-1\right\} > \frac{-\acute{c}_n}{|\acute{a}_n| P_{(R-1)\to R}}$$

and remaining silent if

$$\hat{\Pr}\left\{M_{-n}(t) = R-1\right\} < \frac{-\acute{c}_n}{|\acute{a}_n| P_{(R-1)\to R}}$$

• This simple strategy can be directly played by the node even if, in order to avoid a discontinuous behavior of the players, the adoption of a stochastic version of this solution is typically recommended; this can be defined by means of a smoothed best response as

Node participation strategy

$$\Pr(TX) = \begin{cases} 1 - \frac{e^{-\gamma D}}{2} & \text{if } \hat{\Pr}\{M_{-n}(t) = R-1\} > \frac{-\acute{c}_n}{|\acute{a}_n| P_{(R-1)\rightarrow R}} \\ \frac{e^{\gamma D}}{2} & \text{if } \hat{\Pr}\{M_{-n}(t) = R-1\} \leq \frac{-\acute{c}_n}{|\acute{a}_n| P_{(R-1)\rightarrow R}} \end{cases}$$

which represents two exponential curves connecting at the indifference point; note that is a constant useful to adjust the continuous approximation to the discrete response function.

• It can be proved that even if is not associated with a stable equilibrium, the more profitable solution of the network is always followed given that the adaptation proneness to an environmental conditions change is reduced by a sufficiently large degree of smoothing.

Numerical results

Assumptions:

1) The **game repetition** is run by the packet transmission of a given source node or, in the case of a failed transmission attempt, by the NAK signalling coming from the destination node.

2) **Each link** between a couple of nodes is affected by time-selective Rayleigh fading (the well known Jakes'model has been adopted with normalized Doppler bandwidth $B_d T_u = 10^{-3}$), which, however, can be deemed static during the transmission of each packet. Distinct wireless links are affected by statistically independent fading.

3) The **empirical choices** $T_0 = 5$ and $\beta = 1/10$ have been made.

Numerical results

Assumptions:

4) The **linear models** $f(x) = x$ and $g(x) = k\,x$ have been adopted; the value selected for the parameter k ensures that the two terms range over similar intervals and, consequently, influence the payoffs in a comparable fashion.

5) The **adopted DR-OSTC scheme** is characterized by $L = 15$ and a packet transmission is deemed correct if the involved relay stage reaches a minimum degree of diversity $R = 6$ (all the nodes are supposed to be aware of this value when not differently stated).

6) The **transmission power of the source node** can be adjusted to reach, on the average, a variable number of potential relays. More precisely, assuming a population of **40** potential relays, the power levels $P_{src} = 1$, 2 and 3 allow the source to reach an average number of 15, 30 and 40 nodes, respectively. On the contrary, the **transmission power of the relay nodes** is inversely proportional to the channel attenuation.

Numerical results

• The different strategies we propose are 3:

- strat. #1 (**strat. #2**) is based on the estimation of the number of cooperating nodes in **scenario #1** (**scenario #2**);

- strat. #3 is based on the estimation of the number of selected codewords in **scenario #2.**

• These have been compared with the trivial approach to relay node management proposed in [4] (and dubbed ***always transmit***, AT, in the following); in this approach all the nodes, which are able to correctly decode the packets to be relayed, always forward them towards the destination.

Numerical results

- Numerical simulations have been carried out to assess, for each transmission strategy, the following quantities: 1) its ***average throughput***

$$th \triangleq \frac{N(TX)}{T}$$

where $N(TX)$ is the number of packets correctly transmitted by the whole relay stage in the considered T consecutive time units;

2) its ***energy efficiency*** (*eff*), defined as the mean value, accomplished over the whole set of relay nodes, of the ratio between the overall number of transmission attempts (independently of their success or failure) and the energy spent by the nodes themselves; in other words,

$$eff \triangleq E_{n \in CL}\left(\frac{N(TX_{all}(n))}{E(n,\bar{t})}\right),$$

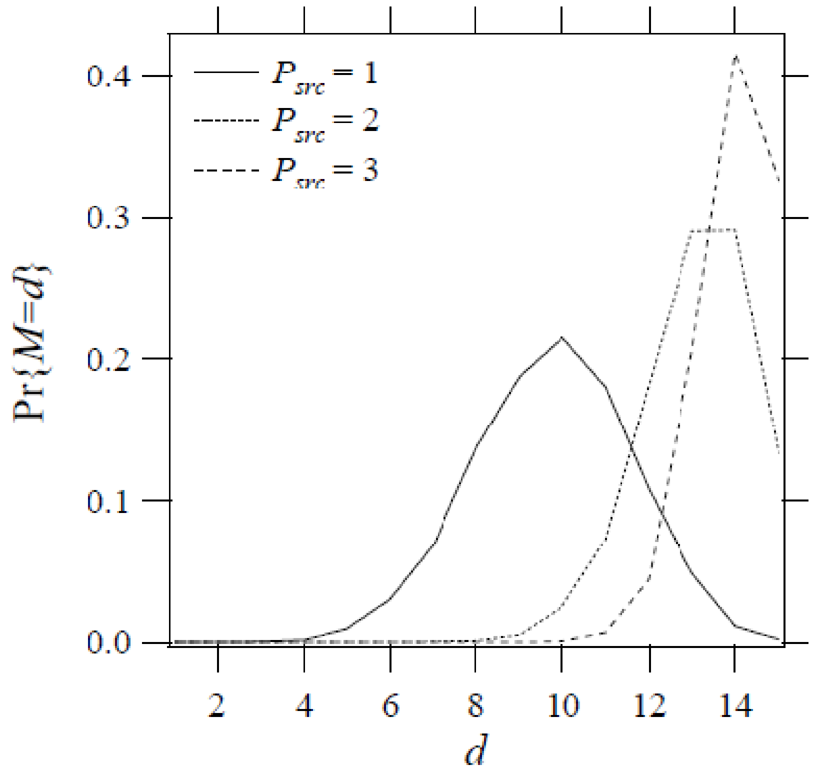
where *n* is the index selecting the node in the cluster *CL* of potential relays, $N(TX_{all}(n))$ is the number of transmission attempts accomplished by the *n*-th node and $E(n,\bar{t})$ is the overall energy spent by the same node until the link closure at the end of the $\bar{t}$-th slot; this energy is evaluated as

$$E(n,\bar{t}) = \sum_{t=0}^{\bar{t}} \frac{1}{|h_{n,t}|^2},$$

where $h_{n,t}$ is the complex channel gain experienced by the *n*-th node towards the destination over the *t*-th time slot.

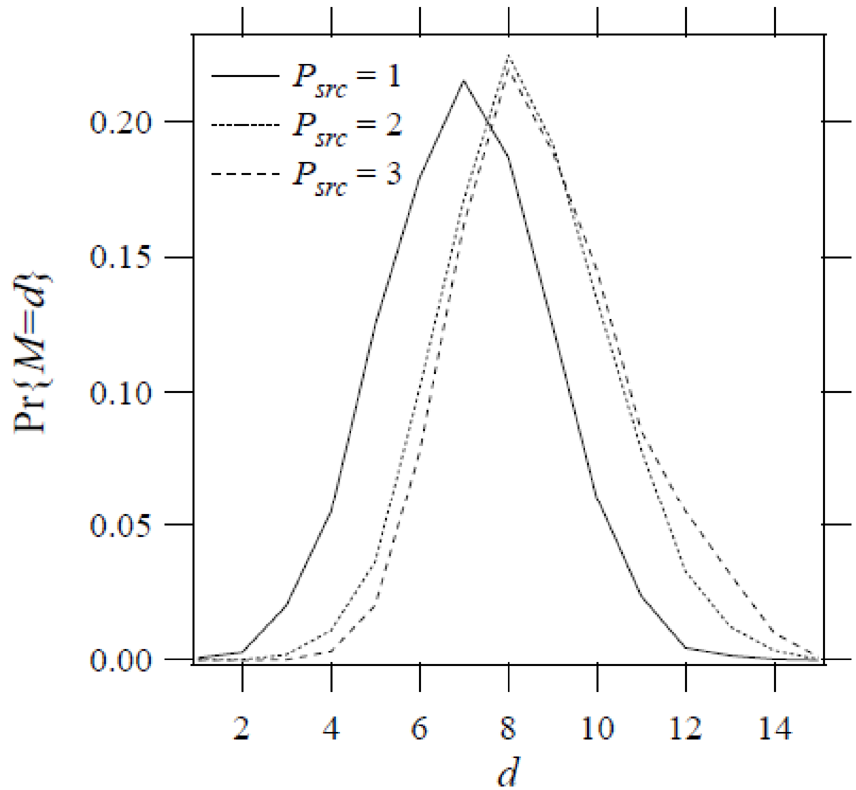
• It is worth noting that the energy efficiency allows to assess the ability of a transmission strategy in exploiting both a limited number of active relays and the best channels within all those available to the potential relays.

Numerical results



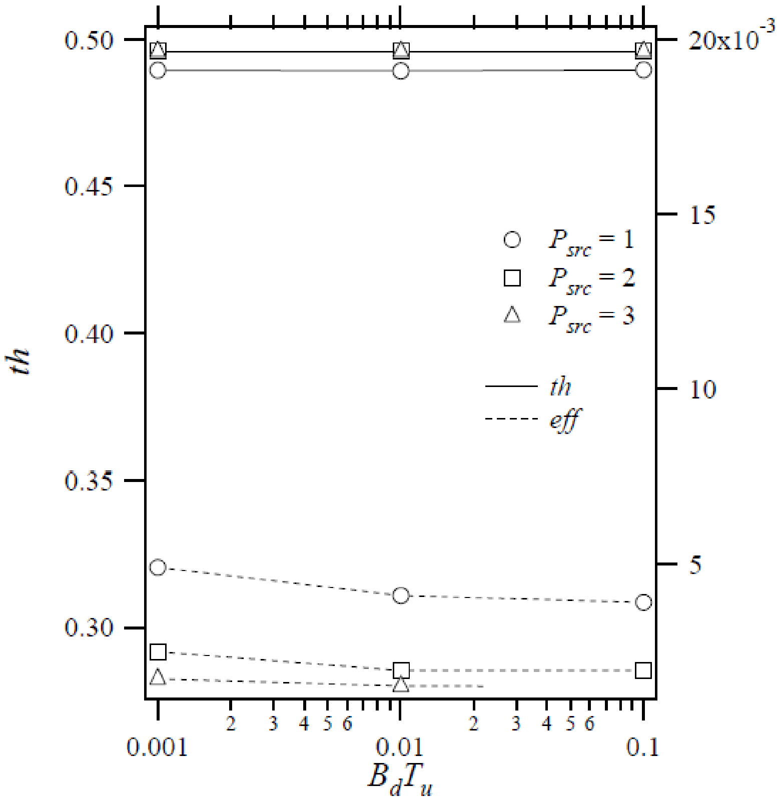
• Estimated probability mass function of the diversity degree achieved by packet transmissions over a link managed in the absence of a control of the cooperating nodes (**AT strategy**).

Numerical results

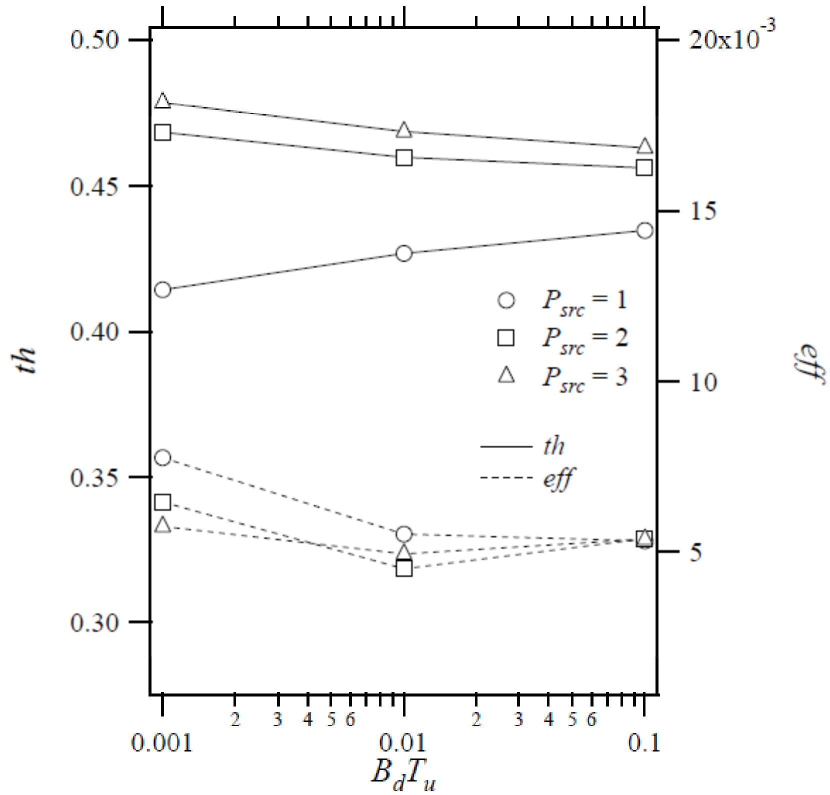


• Estimated probability mass function of the diversity degree achieved by packet transmissions over a link managed in the presence of a control of the cooperating nodes (**proposed strategy**).

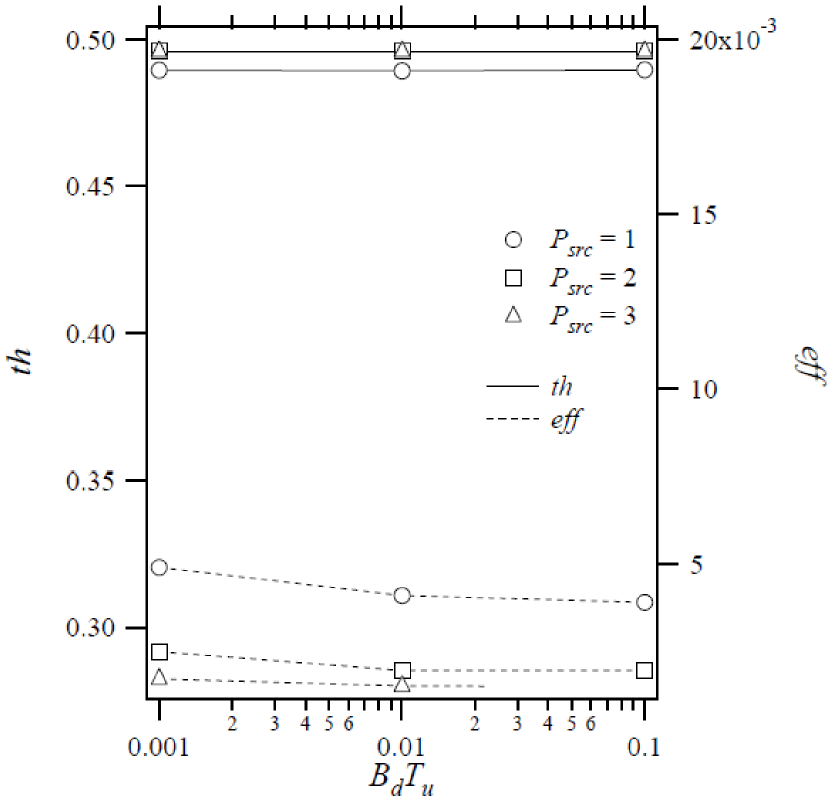
Numerical results



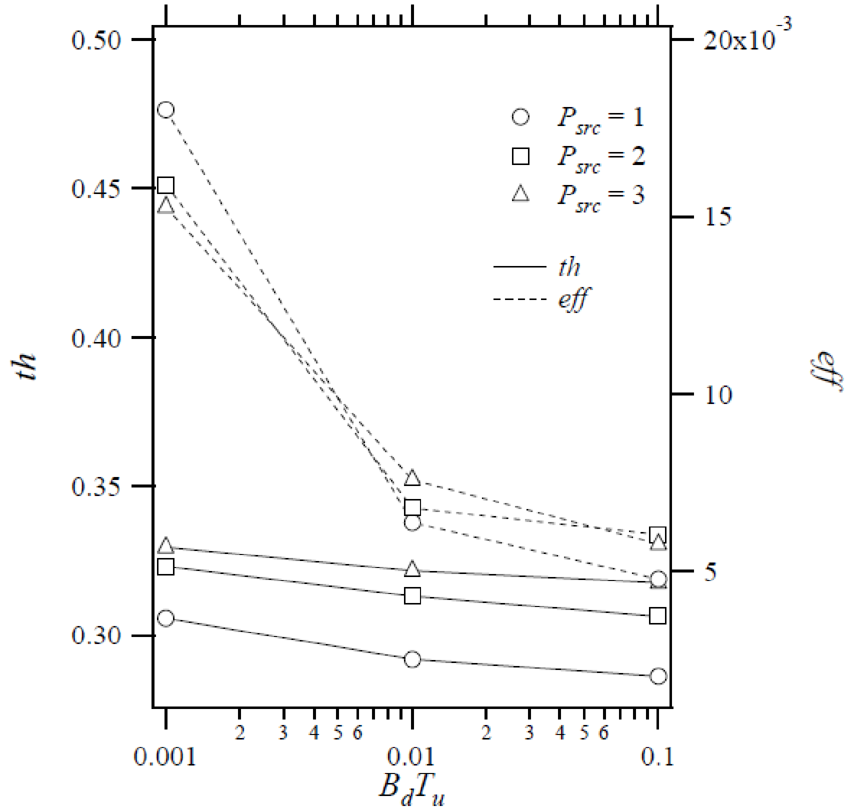
AT strategy



Strategy #1

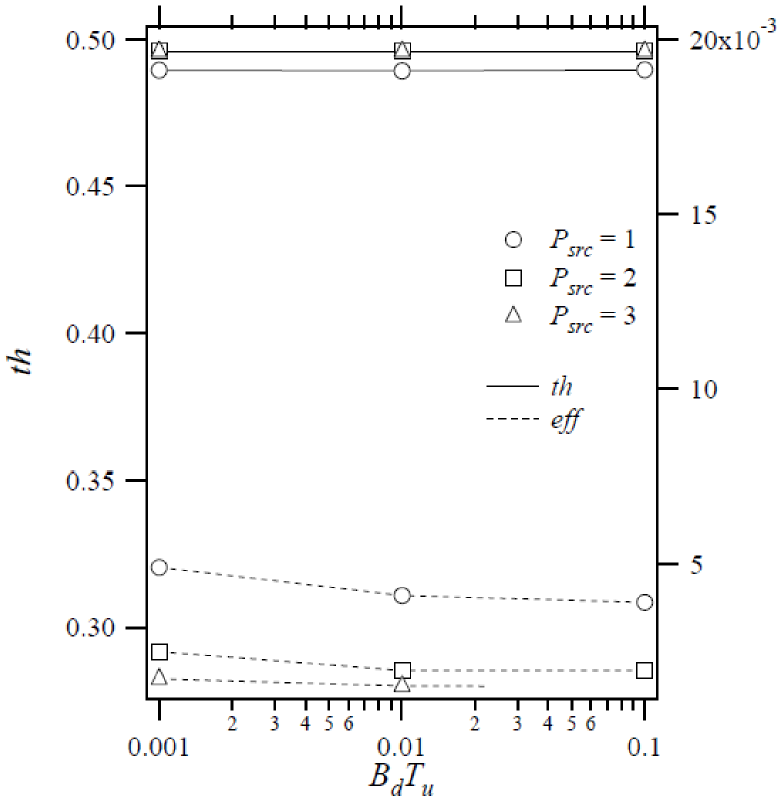


AT strategy

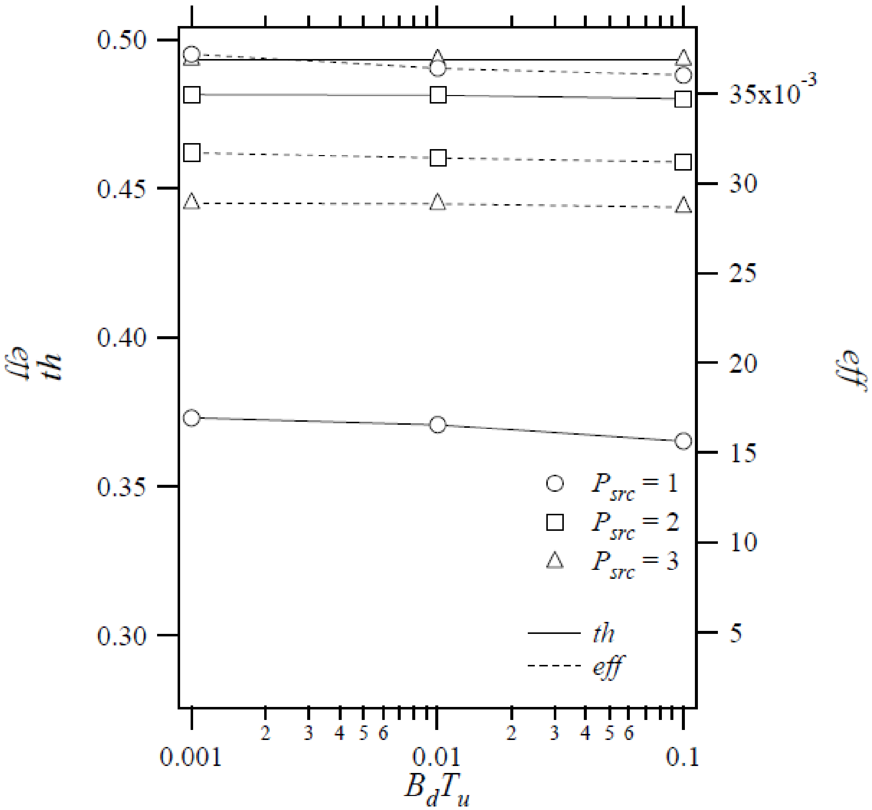


Strategy #2

Numerical results



AT strategy



Strategy #3

Numerical results

• The results about the achievable throughput show that strat. #2 is outperformed by the other two proposed strategies.

• Strat. #1 and strat. #3 offer a throughput similar to that provided by the AT strategy which can be taken as an upper bound for the throughput achievable on the considered relay link.

•If the energy efficiency is taken into consideration, the results are reversed and the performance offered by strat. #2 is slightly better that of strat. #1.

•However, both are outperformed by strat. #3, whereas the AT strategy shows a very low efficiency and, once more, exhibits a performance strongly dependent on the number of nodes able to decode the source transmission.

Numerical results

• Therefore, these results evidence that:

a) The proposed strategies can achieve a superior energy efficiency at the price of a slight decrease in the achievable throughput with respect to a link management ignoring the number of transmitting nodes;

•b) these strategies tend to achieve better performance in the presence a large number of potential relay nodes, i.e. when the AT strategy becomes less efficient.

Numerical results

• The performance difference between strat. #1 and strat. #2 can be explained as follows.

• In the scenario considered for the development of strat. #1, a node can expect a positive payoff from a transmission attempt only if it contributes to a correct transmission accomplished by a cluster; for this reason, even if a large number of nodes within the cluster decides to transmit, a node experiencing a good channel will be prone to transmit too in order to earn a positive (even if low) payoff.

• On the contrary, in the scenario referring to strat. #2, each node earns a fixed payoff irrespective of its contribution in a cluster transmission; therefore it will be less favourable to transmit when it deems that the remaining nodes are able to accomplish the transmission without its contribution, so that an useless energy expense is avoided. This results in a superior energy efficiency of strat. #2, especially for slowly varying channels, at the price of a mean throughput reduction.

Numerical results

• It is also worth pointing out that, in principle, strat. #3 cannot be directly compared with strat. #1 and strat. #2, since it is based on a completely different rationale.

• Despite this, its superior performance can be motivated by its promptness to respond to the system state estimated by each node; in this case, in fact, the best response can be instantaneously played, whereas in strat. #1 and strat. #2 is approached through a continuous adjustment of the transmission probability.

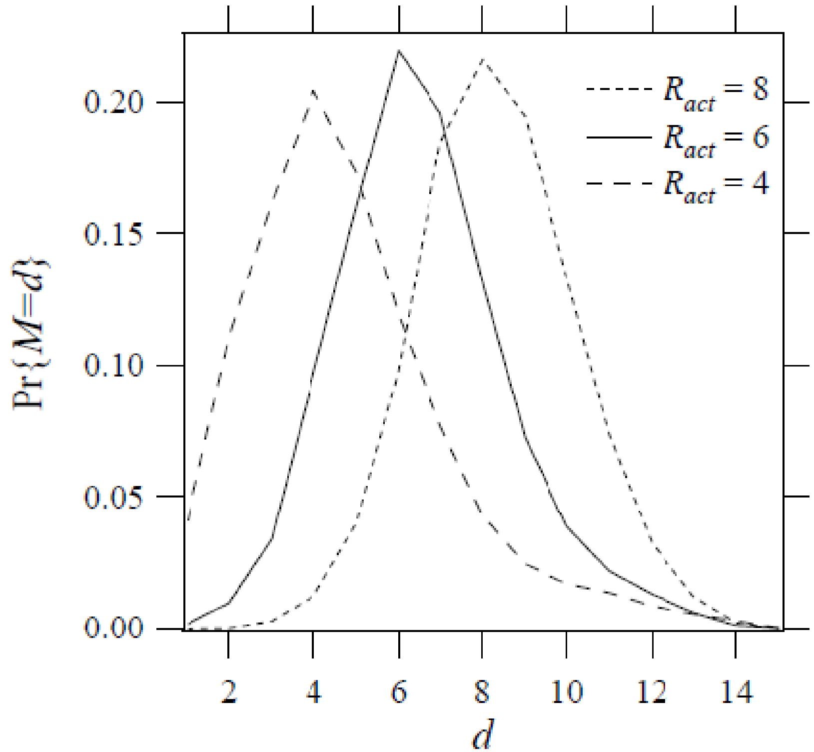
Numerical results

•The performance offered by the proposed strategies in the presence of an imperfect knowledge of the degree of diversity R needed on a cooperative link has been also assessed.

•The following figure illustrates the estimated probability mass function of the degree of diversity achieved by a link when the nodes establishing it compute their strategy under the assumption that *R* = 6, but the degree of diversity actually needed (denoted R_{act} in the figure) is 4, 6 or 8.

•The results referring to the two scenarios characterized by $R_{act} \neq R$ evidence that the proposed approach allows a given cluster to reach a degree of diversity close to that really needed. The low impact of the estimation error represent an interesting property of the proposed solutions. In fact, it allows the cluster behavior to follow the changing needs of a wireless link so that its efficiency is always maximised in the presence of channel variations.

Numerical results



- Estimated probability mass function of the diversity degree achieved by a link managed in the presence of a control of the involved nodes when R = 6 and R_{act} = 4, 6, 8.

Conclusions

• The problem of node management in cooperative data transmissions based on a DR-OSTC scheme has been discussed.

• Some solutions have been developed resorting to various tools provided by game theory. The devised strategies for node management are *fully distributed*, since are characterized by autonomous choices made by each potential relay node

• They allow to coordinate the transmissions among the potential relays *without any explicit information exchange among them*, so avoiding the drawback of transmission overhead.

• The proposed solutions are of significant practical interest since they allow, on the one hand, to guarantee the participation of a proper number of nodes to a transmission cluster (hence avoiding an energy waste associated with an excessive number of transmissions) and, on the other hand, to avoid a considerable throughput decrease with respect to unmanaged solutions.

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